SRI GCSR DEGREE COLLEGE

DEPARTMENT OF PHYSICS

III B.Sc. Semester-IV

Modern Physics

PHYSICS PAPER-V

Prepared by
G.M.Murali Krishna

HOD (Dept. of Physics)

A.Lakshman Rao

Sr Lecturer in Physics

SYLLABUS

UNIT-I :

1. Atomic and Molecular Physics:(12 hrs) Vector atom model and Stern-Gerlach experiment, Quantum numbers associated with it, Angular momentum of the atom, Coupling schemes, Spectral terms and spectral notations, Selection rules, Intensity rules, Fine structure of Sodium D-lines, Zeeman effect, Experimental arrangement to study Zeeman effect; Raman effect, Characteristics of Raman effect, Experimental arrangement to study Raman effect, Quantum theory of Raman effect, Applications of Raman effect.

UNIT-II:

2. Matter waves & Uncertainty Principle: (12 hrs) Matter waves, De Broglie Hypothesis Wave length of matter waves,properties of matter waves,Davisson and Germer experiment, Phase and group velocities, uncertainty principle for position and momentum& energy and time, Illustration of uncertainty principle using diffraction of beam of electrons (Diffraction by a single slit)and photons(Gamma ray of complementarily).

UNIT-III:

3. Quantum (Wave) Mechanics:(12 hrs) Basic postulates of quantum mechanics, Schrodinger time independent and time dependent wave equations-Derivations, Physical interpretation of wave function, Eigen functions, Eigen values, Application of Schrodinger wave equation to (i) one dimensional potential box of infinite height(Infinite Potential Well) and (ii) one dimensional harmonic oscillator

UNIT-IV:

4. Nuclear Physics:(12 hrs) Nuclear Structure: General Properties of Nuclei, Mass defect, Binding energy; Nuclear forces: Characteristics of nuclear forces- theory; Yukawa's meson theory,Nuclear Models: Liquid drop model, The Shell model, Magic numbers; Nuclear Radiation detectors: G.M. Counter, Cloud chamber, Solid State detector; Elementary Particles: Elementary Particles and their classification

UNIT-V:

5. Nano materials:(7hrs) Nano materials Introduction, Electron confinement, Size effect, Surface to volume ratio, Classification of nano materials (0D, 1D, 2D); Quantum dots, Nano wires, Fullerene, CNT, Graphene (Mention of structures and properties),Distinct properties of nano materials(Mention-mechanical, optical, electrical, and magnetic properties); Mention of applications of nano materials: (Fuel cells,Phosphors for HD TV, Next Generation Computer chips, elimination of pollutants, sensors)

6. Superconductivity: (5 hrs) Introduction to Superconductivity, Experimental results-critical temperature, critical magnetic field, Meissner effect , Isotope effect, Type I and Type II superconductors, BCS theory (elementary ideas only),Applications of superconductors

AdiKavi Nanayya University::Rajamahendravaram
III B.Sc. : Physics Paper VI Semester-V
Modern Physics
Model Paper

TIME: 3Hrs   Max. Marks: 75

SECTION-A

Answer any FIVE questions   5x5=25M

1. Explain L-S coupling and J-J coupling.

   L-S సమ్మేళనము మరియు J-J సమ్మేళనములను గూర్చి వివరించండి.

2. Explain Raman Effect and symmetry of Raman lines on the basis of quantum theory.

   క్వాంటం సిద్ధాంతము ఆధారంగా రామన్ ఫలితాన్ని రామన్ రేఖల సౌష్టవమును వివరించండి.

3. Explain liquid drop model.

   ద్రవ బిందు నమూనాను వివరించుము.

4. Explain miller-indices.

   మిల్లర్ సూచికలు గూర్చి వివరముగా తెల్పండి.

5. Explain Basic Postulates of Quantum Mechanics.

   క్వాంటమ్ సిద్ధాంతము యొక్క ప్రతిపాదనను వివరించండి

6. Calculate the de-Broglie wave length associated with a proton moving with a velocity of 2200m\sec.(h=6.625×10^{-34}J-S,m=1.6×10^{-27}kg)

   2200m\sec వేగంతో చలిస్తున్న ప్రోటాన్ యొక్క డీబ్రోగ్లీ తరంగ ధైర్ఘ్యంను లెక్కించుము.

   (h=6.625×10^{-34}J-S, m=1.6×10^{-27}kg)

7. A neutron breaks into a proton and an electron. Calculate the mass defect in the reaction. (m_P=1.6725×10^{-27}Kg ,m_e =9×10^{-31}kg,m_n=1.6747×10^{-27}Kg)

   ఒక కేంద్రక చర్యలో ఒక న్యూట్రాన్ ప్రోటాన్ మరియు ఎలక్ట్రాన్ గా విడిపోయింది. ద్రవ్యరాశి లోపం లెక్కించుము.

   (m_P=1.6725×10^{-27}Kg , m_e =9×10^{-31}kg,m_n=1.6747×10^{-27}Kg)

8. X-rays of wave length 1.54A^0 are diffracted by a crystal. The incident angle 11^0, what is the lattice space distance.

   స్ఫటికం వలన తరంగ ధైర్ఘ్యం 1.54A^0 కలిగిన X-

   కిరణాలు వివర్తనం చెందినది. పతనకోణం 11^0 జాలక అంతర దూరమును కనుగొనుము.

SECTION-B

Answer any ALL questions   5x10=50M

9. (a) Describe stern and Gerlach experiment. What is its importance?

   స్టెర్న్–గెర్లాక్ ప్రయోగాన్ని వివరించండి. ఆ ప్రయోగ ప్రాముఖ్యత ఏమిటి?

   (or)

   (b)What is Raman Effect? How it is experimentally studied.

   రామన్ ఫలితం అనగానేమి? దానిని ప్రయోగ పూర్వకంగా వివరించండి.

10. (a) Explain De-Broglie hypothesis for matter waves. Derive an expression for De-broglie wave length.

ద్రవ్య తరంగముల గురించి డీ బ్రోగ్లీ పరికల్పన వివరించండి. డీ–బ్రోగ్లీ తరంగ దైర్ఘ్యమునకు సమీకరణం ఉత్పాదించుము.

(or)

(b) Explain Devison and Germer experiment for detection of matter waves.

ద్రవ్య తరంగాలను శోధించుటకు డెవిజన్ గెర్మర్ ప్రయోగాన్ని వివరింపుము.

11. (a) Derive Schrödinger's time independent wave equation.

కాలం మీద ఆధార పడని ష్రోడింగర్ తరంగ సమీకరణమునకు ఉత్పాదించుము.

(or)

(b) Obtain an expression for the energy of a particle in one dimensional potential wall.

ఏక నిరూపక పొటెన్షియల్ కూపములోని కణము యొక్క శక్తికి సమీకరణము రాబట్టండి.

12. (a) Explain magic numbers using nuclear shell model.

కేంద్రక కర్పర నమూనాను ఉపయోగించి మ్యాజిక్ సంఖ్యలను వివరించండి?

(or)

(b) Explain Gamow theory of α –decay.

α-కణ క్షీణత విషయంలో గేమో సిద్ధాంతమును వివరించుము.

13. (a) Describe laue method for the study of crystal structure.

స్పటిక నిర్మాణకు సంబంధించి లవే ప్రయోగాన్ని వర్ణించండి?

(or)

(b) What is super conductivity? Explain Meissner effect. Mention the properties of super conductivity.

అతి వాహకత్వం అనగానేమి? మిస్నర్ ఫలితాన్ని వివరింపుము.అతి వాహకాల ధర్మలను తెల్పుము.

Blue Print

Module	Essay Questions 10 marks	Short Questions 5 marks	Marks allotted
1. **Unit - I**	2	2	30
2. Unit - II	2	1Problem	25
3.Unit - III	2	1	25
4.Unit -IV	2	1+1Problem	30
5.Unit - V	2	1+1Problem	30
Total			140

UNIT – I ATOMIC & MOLECULAR PHYSICS

Q.1) What are the drawbacks of Bohr's Theory? or explain the limitations of Bohr's Theory.

A) 1. The theory could not explain more complex spectra of atoms than hydrogen.

2. Does not give any information on distribution and arrangement of electrons in atom.

3. Does not explain variations in intensity of spectral lines of an element.

4. Unable to calculate the rate of transitions and selection rules applicable to them.

5. Fails to account for fine structure of spectral lines

6. Fails to account for Zeeman and Stark Effects.

7. Unable to give quantitative explanation of chemical bonding.

Q.2) Describe the concept of Vector Atom Model.

A) The two concepts which characterize the Vector atom model and differentiate from other models are:

1. The Concept of Space Quantization and
2. The Concept of Spinning electron

1. **Space Quantization**

According to Bohr theory, electron has only one degree of freedom as it moves in a constant circular orbit with radius fixed. The principal quantum number n specifies it.

According to Sommerfeld's model, electron possess two degrees of freedom as it moves in an elliptical orbit. These are the two quantum numbers n_r corresponding to radius and n_φ corresponding to azimuthal angle.

In general, an electron moves in three dimensional space hence one more degree of freedom is required. This gives rise to the assumption of quantization of orbits both in magnitude and direction. According to the rule of space quantization, electron can orient itself only in certain discrete direction with respect to applied magnetic field direction. This is known as Space Quantization and was proposed by Sommerfeld.

2. **Spinning Electron**

Uhlenbeck and Goudsmit put forward their hypothesis of electron spin.

According to this hypothesis, the electron revolves not only in an orbit around the nucleus but also about its own axis. Thu electron has two types of motion i.e. Orbital and Spin motion. Hence the angular momentum of the atom will be due to both orbital and spin motion of electron.
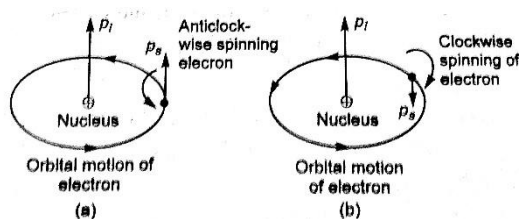
According to Quantum theory, spin motion like orbital motion is also quantized. Hence a new quantum number, known as spin quantum number is introduced. Both orbital and spin motions are quantized not only in magnitude but also in direction according to concept of space quantization. The two are considered as quantized vectors and hence the atom model is called a Vector Atom Model.

Q.3) Describe the Stern and Gerlach experiment and discuss how it verifies the concept of Space quantization and electron spin. (OR) Describe the Stern and Gerlach experiment and indicate the importance of the results Obtained.

A) **Experimental Arrangement :** In Stern and Gerlach experiment a beam of silver atom is passed through an inhomogeneous magnetic field. In the figure, O is an electric oven which on heating produces the silver atomic beam. It is then passed through two narrow slits S_1 and S_2.
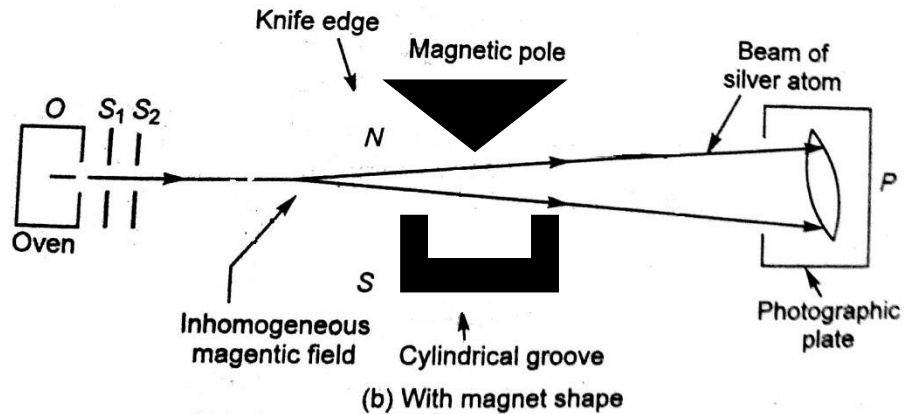

(b) With magnet shape

Fig. Stern-Gerlach experiment.

The inhomogeneous magnetic field is produced by taking the pole pieces of the magnet as: knife edge shaped North pole above and a flat groove shaped South pole below respectively. As we move from the center towards the knife edge N pole, intensity of magnetic field increases, while it decreases we go below towards the flat groove S pole. A photographic plate P records the pattern of the beam after passing through the field. In the absence of the field, a narrow strip is formed as shown in the figure (a) below. In the presence of the field, the strip splits two components as shown in the figure (b). But the classical pattern is as shown in the figure (c).
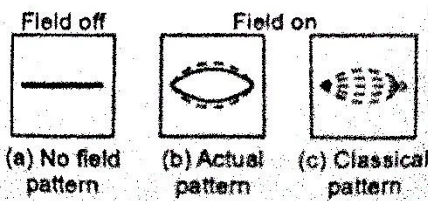
The splitting of silver beam into two components verifies the existence of Space quantization and electron spin.

Expression for Displacement of the Silver atomic beam along Field direction

Suppose the magnetic field be non-homogeneous along Y-direction. The field gradient is dB/dy and is positive.

Let the atomic magnet with magnetic moment M, pole strength p and length l incline at an angle θ with the field direction and let it be placed in such a field as shown in fig. (16). If the field strength at one pole is B, then the field strength at other pole will be $B + (dB/dy)\, l\cos\theta$.
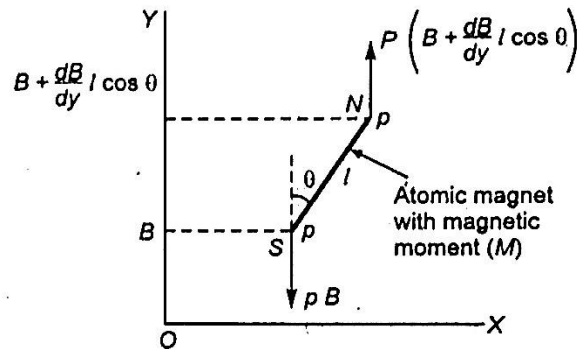

Fig. (16)

Force on one pole of atomic magnet is $p\,B$ while on other pole is $p\,(B + dB/dy \cdot l\cos\theta)$. The extra force $p\,dB/dy \cdot l\cos\theta$ on one pole displaces the atom as a whole. The force F_y is given by

$$F_y = p\cdot l\cos\theta \cdot \frac{dB}{dy} = M\cos\theta\,\frac{dB}{dy} \qquad\qquad ...(1)$$

where $p\,l = M$ = Magnetic moment.

Due to this force, the atomic magnet will be displaced from its straight path in the field direction. To find out this displacement, let the silver atom enter the non-homogeneous field with velocity v and the length of path be L for which time taken is t.

The displacement $(d)_y$ of the atom along the field direction at the end of time t is given by

$$(d)_y = \frac{1}{2}\alpha_y\, t^2 \qquad\qquad ...(2)$$

where α_y is the acceleration imparted to the atom along the field direction by force F_y. Thus, $\alpha_y = F_y/m$ where m is the mass of atomic magnet.

$$\therefore \qquad (d)_y = \frac{1}{2}\left(\frac{F_y}{m}\right)\cdot t^2 = \frac{1}{2}\frac{F_m}{m}\left(\frac{L}{v}\right)^2 \qquad \left(\because t = \frac{L}{v}\right)$$

Sustituting value of F_v from eq. (1), we get

$$(d)_y = \frac{1}{2}\frac{M\cos\theta}{m}\frac{dB}{dy}\left(\frac{L}{v}\right)^2$$

TUsing this the displacement of the silver atomic beam can be found. The experimental and theoretical results are in excellent agreement; hence this provides verification of   Space quantization and electron spin.

Q.4) Explain various Quantum Numbers associated with the Vector Atom Model. A)
The Quantum Numbers associated with the electron in a given atom are:
   1) **Principal Quantum Number (n)**
   2) **Orbital Quantum Number (l)**
   3) **Spin Quantum Number (s)**
   4) **Magnetic Orbital Quantum Number (m_l)**

1) **Principal Quantum Number (n):** This quantum number represents the Orbit number or main energy level or shell of the electron. n takes only integer values 1, 2, 3.......The orbit with n=1 is called 1st Orbit or K-Shell or K main energy level, n=2 represents 2nd Orbit or L-Shell or L main energy level and so on. This determines the size of the orbit and energy of electron in that orbit.

radius of nth orbit $r_n \propto n^2$ and energy of electron in n^{th} orbit $E_n \propto -1/n^2$

2) **Orbital Quantum Number (l):** This quantum number defines the shape of the Orbital occupied by the electron and hence known as Orbital or Angular quantum number. It takes values 0 to (n-1), each representing an energy sub shell or sub shell or orbital. Orbitals corresponding to l=0,1,2,3,4...... are designated as *s, p, d, f* etc $p_l = l\, h/2\pi$ Also this governs the degree to which electron is attached to nucleus.

3) **Spin Quantum Number (s):** The electron not only has orbital motion around the nucleus but also rotates about its own axis i.e. it has spin motion. According to quantum theory, angular momentum due to spin motion is: $p_s = s\dfrac{\hbar}{2\pi}$ where s is called Spin quantum number. S takes only two values $\pm\dfrac{1}{2}$.

4) **Magnetic Orbital Quantum Number (m$_l$):** The projection of *l* vector in the direction of external applied magnetic field is known as magnetic orbital quantum number. This takes only values –*l* to +*l* through zero. Thus for a given value of *l*, possible values of **m$_l$** are given as: –*l*, -(*l-1*), -(*l-2*) ....-1, 0, 1, 2, ...(*l-2*), (*l-1*), *l*.
For example, for *l*=1 **m$_l$** has 3 orbitals given as -*l*, 0, +*l*.

5) **Magnetic Spin Quantum Number (m$_s$):** The projection of *s* vector in the direction of external applied magnetic field is known as magnetic spin quantum number. Since *s* takes only two values, also (**m$_s$**) takes only two v$\pm\dfrac{1}{2}$es

6) **Total Angular Momentum Quantum Number (J):** The vector sum of orbital angular momentum and spin angular momentum of the electron is called its total angular momentum. According to $p_J = j\left(\dfrac{h}{2\pi}\right)$ quantum theory and J=*l*±s

7) **Magnetic Total Angular Momentum Quantum Number (m$_J$):** The projection of total angular quantum number J on the direction of magnetic field is known as

magnetic total angular momentum quantum number. It takes the values from –J to +J including 0. Thus there are (2j+1) values of m_J

Q.5) Explain various Coupling Schemes. (or) What are L-S and J-J Couplings? Explain.

   A) Generally two types of Coupling occur namely: Russel – Saunders Coupling or L-S Coupling and J-J Coupling respectively, described as below:

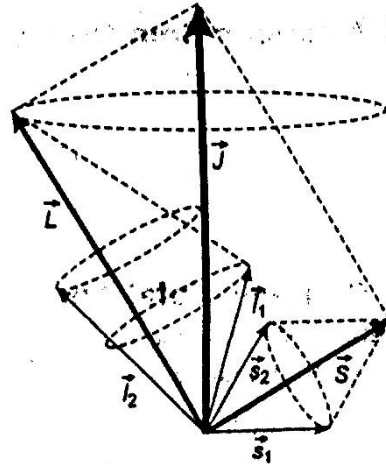1. **L-S Coupling:** This type of coupling occurs most frequently and hence this is known as Normal Coupling. In this type of coupling, all the Orbital angular momentum vectors *l* of the electrons combines to form a resultant vector **L** and all Spin angular momentum vectors likewise combine to form a resultant vector **S**.
   Now these Vectors L & S combine to form Vector **J** which represents the total angular momentum of the atom. Symbolically, we represent the coupling as:

   L = (l_1+l_2+l_3+.......)
   S = (S_1+ S_2+ S_3+......)
   And **J = L + S** that can be shown as in the fig.

2. **L-S Coupling:** This type of coupling occurs in some cases where interaction between l and s vectors in each electron is stronger than interaction among either of each. In this coupling each electron is considered separately and the total angular momentum J is obtained from the relation: ***j = l + s***
   The total angular momentum **J** of the atom is equal to the vector sum of all the individual j vectors of the electrons. ***j_1 = (l_1 + s_1), j_2 = (l_1 + s_1) j_3 = (l_3 + s_3)** ....
   and thus ***J = (j_1+ j_1+ j_1)** +............ = Σ ***j***
   Pure J-J coupling is rarely found. In most cases, L-S is effective. This is shown in fig.
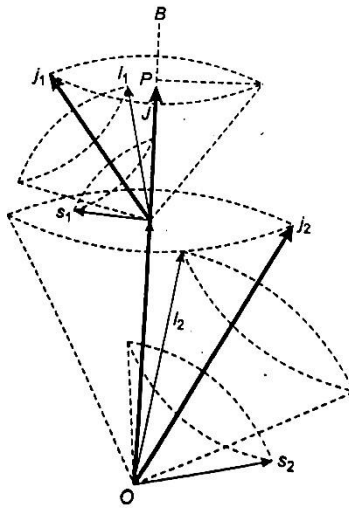
Q.6) What is Raman Effect? Describe the experimental arrangement (or Setup) to study Raman Effect.

A) Raman Effect: Whenever a monochromatic beam of light is passed through a gas, liquid and a transparent solid, a fraction of light is scattered in all directions. The spectrum of scattered light consists of three frequencies one the incident frequency along with a frequency greater and a frequency lesser than incident. This was observed by Sir C.V. Raman and hence known as the Raman Effect.

The spectrum of scattered light is called Raman Spectrum and the new lines are known as Raman Lines. The lines of greater frequency are called as **anti-stokes lines** while the lines of smaller frequency are called **stokes-lines**. These are shown in the figure below:
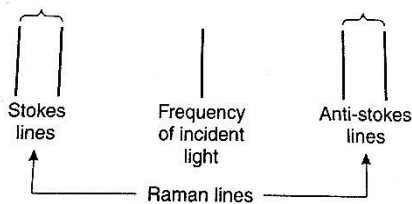
Characteristics of Raman Effect:

1) Raman Effect is purely Molecular Phenomena

2) Raman lines are characteristics of scattering material

3) Raman lines are strongly polarized

4) Raman lines are symmetrically placed about parent line or un-modified line

5) Frequencies of Raman lines depend on incident frequency

Experimental arrangement of RAMAN EFFECT

The experimental arrangement consists of three main parts namely: (1)
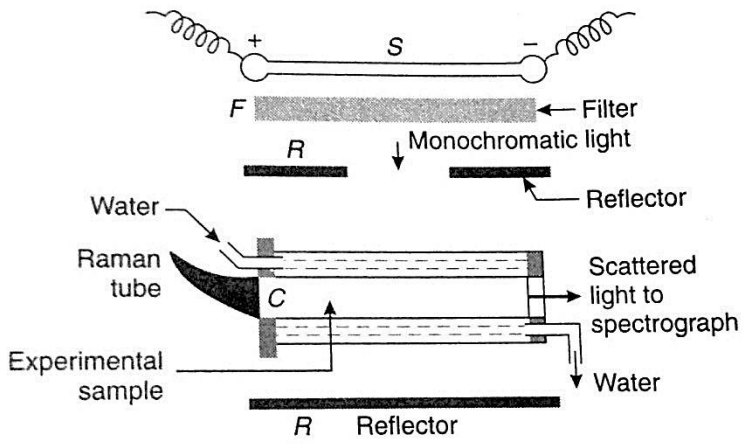**Source,** (2) **Raman Tube and** (1) **Spectrograph**

Fig. (30)   **Experimental arrangement for Raman spectrum**

In the figure above, S is a Mercury arc lamp which acts as a source of light. Light from it is passed through a filter that produces monochromatic light. This is then passed through a small opening of metallic reflector to fall on Raman tube.

Raman tube is a glass tube of 1 or 2 cm diameter and length 10or 15 cm. One end of it is a flat glass surface from which scattered light emerges. Other end is shaped into a horn and is blackened outside to act as black background.

The experimental sample (gas or liquid or transparent solid) for which Raman spectrum is to be studied is placed inside the Raman tube. The tube is surrounded by a water jacket that acts as cooling system in preventing overheating of sample due to nearness of arc lamp. The scattered beam is allowed from flat end and is studied by a spectrograph.
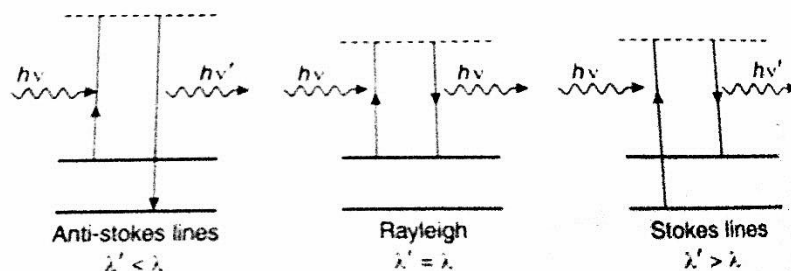
Q.7) Explain Quantum theory of Raman Effect.

A) Prof. Smekal in 1929 explained the Raman Effect using Quantum Theory. According to this theory, the light Souce emits photons of energy $h\nu_0$. When this photon hits a molecule of sample 3 things might take place:

(1) There will be an Unmodified line when photon does not interact with the molecule.

(2) May be if a photon hits excited molecule, then molecule donates part of its energy to photon resulting in its decrease (E_1-E_2). Now the photon energy is $h\nu_0+(E_1-E_2)$. The scattered radiation frequency thus increases, which corresponds to Anti Stokes line frequency, given by: $\nu_{a.s} = [h\nu_0+(E_1-E_2)]/h = \nu_0+(E_1-E_2)/h$

(3) May be if excited photon hits the molecule, then molecule gains energy from it so that photon loses a part of its energy in exciting the molecule. Now photon energy is $h\nu_0-(E_1-E_2)$. Thus scattered radiation frequency decreases, which corresponds to Stokes line frequency, given by:

$\nu_s = [h\nu_0-(E_1-E_2)]/h = \nu_0-(E_1-E_2)/h$ In this way three lines are obtained in the Spectrum.

Anti-stokes lines   Rayleigh   Stokes lines
$\lambda' < \lambda$ $\lambda' = \lambda$ $\lambda' > \lambda$

Q.8) Write the Applications of Raman Effect or Give the Applications of Raman Effect.

A) Following are some of the applications of Raman Effect:

1) To study molecular structures of crystals and molecules

2) To study composition of plastics and mixtures

3) To know single, double or triple bond

4) To study spin and statistics of nuclei

5) To study bond length, bond strength and bond angle

6) To know number of atoms in a molecule

7) To study binding forces in molecules, crystals

8) To study vibrational and rotational energy levels of molecules

Q.9) What is Zeeman Effect? or Define Zeeman Effect.

A) The splitting up of spectral lines of an atom in the presence of an external applied magnetic field is known as Zeeman Effect.

If the magnetic field is very strong, then each spectral line splits up into two components in the longitudinal view and three components in transverse view. This is known as Normal Zeeman effect. When the magnetic field is weak, then each line splits up into more than three components and this is known as Anomalous Zeeman Effect.

UNIT – II

MATTER WAVES & UNCERTAINITY PRINCIPLE

Q.1) Explain de-Broglie Hypothesis of Matter Waves and Derive the expression for de-Broglie Wavelength (or) Describe de-Broglie Hypothesis and Derive the expression for Wavelength of matter waves

A) 1. Matter and Radiation were first assumed to consist of particles, but phenomena like interference, diffraction and polarization showed that radiation is composed of waves.

2. Wave theory could not explain Photoelectric Effect, Compton Effect etc. But were explained on the basis of Quantum theory assuming photons as particles. Due to these consequences, Light is attributed Dual nature i.e.it possess both particle and wave properties.

3. Louis de-Broglie extended this Wave – Particle Duality not only to Electromagnetic radiation (light) but also to Matter. This suggestion was based on the fact that Nature Loves Symmetry.

2. According to **de-Broglie Hypothesis**, a moving particle is associated with a wave known as **de-Broglie wave or matter wave**, whose wave length is given by:

$$\lambda = h/mv = h/p$$

Hence in conclusion:

a. Waves and Particles are two modes of energy transfer in nature.

b. Universe is entirely composed of Matter and Radiation.

c. Nature loves symmetry. Hence matter and waves must be symmetrical i.e. they both act like particles sometimes and as waves at other times.

de-Broglie Wavelength of Matter Waves

The expression for wavelength associated with a material particle can derived from the similarity of radiation as follows:

From Planck's Quantum theory of radiation, the energy of a photon is given by:

$$E = h\nu = hc/\lambda \qquad ----- 1$$

Where c is the velocity of light in vacuum and λ is its wavelength.

According to Einstein's Mass-Energy relation, $E = mc^2$ $\qquad$ ------- 2

From equations (1) and (2), we get:

$$mc^2 = hc/\lambda \qquad \lambda = hc/mc^2 \qquad \lambda = h/mc \text{ where}$$

p=mc is the momentum of photon.

In the case of a Material particle of mass m, moving with a velocity v, having momentum p=mv, the wavelength associated is given by:

$$\lambda = h/mv = h/p$$

Q.2) Explain the Properties of Matter Waves (or) Give Properties of Matter Waves A)

Following are the Properties of Matter Waves:

1. Lighter is the particle, greater is the wavelength associated.
2. Smaller the velocity, greater is the wavelength associated.
3. When v=0 then wavelength $\lambda=\infty$ i.e. wave becomes indeterminate and if $v=\infty$ then $\lambda=0$. This shows that matter waves are generated by the moving particles.
4. The velocity of matter waves depends on the velocity of matter particle, i.e., it is not a constant while velocity of a EM wave is constant.
5. The velocity of matter wave is greater than velocity of light. $\qquad$ This is proved as follows:

We know that $\qquad\qquad$ $E = h\nu$ and $E = mc^2$

$\Rightarrow \qquad\qquad h\nu = mc^2$

$\Rightarrow \qquad\qquad \nu = mc^2/h$

Matter wave velocity is given by: $\omega = \nu \times \lambda = (mc^2/h) \times (h/mv)$

So $\therefore \omega = c^2/v$

Clearly this shows that the Matter waves travel at a speed much greater than the speed of light

6. The Wave and Particle aspects of moving bodies never appear together in the same experiment i.e. both are complimentary but not contradictory.
7. The Wave nature of matter introduces an uncertainty in the location of position of particle

Q.3) Explain Davisson and Germer Experiment for the detection of Matter Waves (or) Describe Davisson and Germer Experiment on electron diffraction and Discuss its results.

A) The first experimental evidence of Matter Wave was given by Davisson and Germer in 1927. They also measured the de- Broglie wavelength of slow moving electrons. **Experimental arrangement**
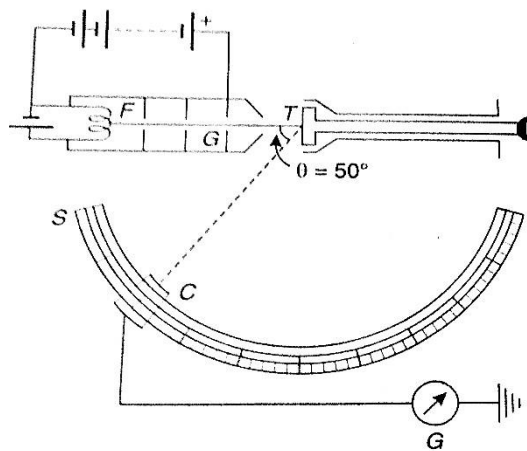
The Experimental arrangement consists of an Electron Gun *G* from which a pencil electronic beam is obtained by using a Filament F heated to dull red. Now electrons are accelerated in an electric field of known P.D. After this electron beam is collimated by using suitable slits.

Beam is allowed to fall on a Nickel Target T. The electrons from these acts as waves diffracted.

An electron detector namely Faraday Cylinder C measures the angular distribution, which is connected to a galvanometer. The cylinder can be moved on a circular graduated scale S between 29º to 90º. **Procedure**

Setting the crystal at some angle, an accelerating potential is applied from a low value. The Cylinder C is then moved on scale S and correspondingly galvanometer reading is noted. The galvanometer deflection forms the measure of electrons Intensity.

A graph is plotted between galvanometer current and Ө for different accelerating potentials. The corresponding curves are as shown below:
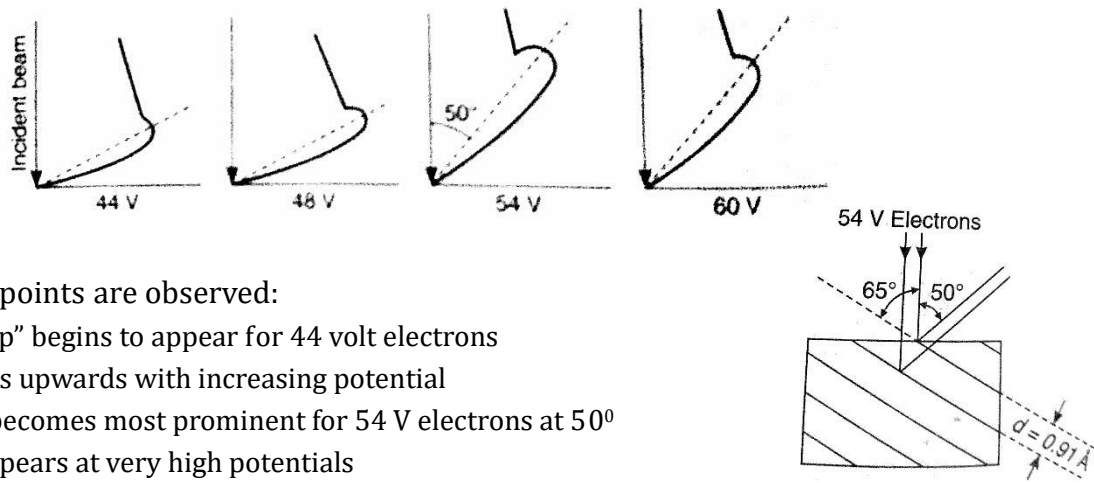
Following points are observed:
1) A "bump" begins to appear for 44 volt electrons
2) It moves upwards with increasing potential
3) Bump becomes most prominent for 54 V electrons at 50^0
4) It disappears at very high potentials
Observations

As bump in the most prominent state verifies electron waves, the de-Broglie wavelength associated with the 54 V electrons is : $\lambda = 12.26/\sqrt{54} = 1.67$ Å **Confirmation**

The Nickel Crystal acts as plane diffraction grating with spacing d=0.91 Å and Using Bragg's Equation (taking $\theta = 65^0$, n=1) we have $\lambda = 2d \sin \theta = 2 (0.91 Å) \sin 65^0 = 1.65$ Å

As the two values are in good agreement, hence confirm the de-Broglie Concept of Matter Waves.

Q.1) State & Explain Heisenberg Uncertainty Principle for P and x. Extend it to energy & time
A) According to Heisenberg Uncertainty Principle, it is impossible to measure both the position and momentum of a particle simultaneously to any desired degree of accuracy.
In general, this principle states that "It is impossible to specify precisely and simultaneously the values of conjugate pairs of physical variables that describe the behaviour of an atomic system". Also qualitatively this states that: The Product of Uncertainties in two physical variables will always lie in the order of magnitude of Planck's constant. Considering the pair of physical variables as position and momentum, we have:

$$\Delta p \, \Delta x \approx h$$

Where Δp is the uncertainty in determining momentum and Δx is the uncertainty in determining the position of the particle. Similarly, we have:

$$\Delta E \, \Delta t \approx h \qquad \text{and} \qquad \Delta J \, \Delta \theta \approx h$$

Where ΔE and Δt are the uncertainties in determining energy & time while ΔJ and $\Delta \theta$ are the uncertainties in determining angular momentum and angular displacement or angle.
The most exact statement of Heisenberg principle is: The Product of uncertainties in determining the position and momentum of the particle can never be smaller than the order of $h/4\Pi$.
So we have: $\Delta p \, \Delta x \geq h/4\Pi$, $\Delta E \, \Delta t \geq h/4\Pi$, $\Delta J \, \Delta \theta \geq h/4\Pi$ respectively.
TIME – ENERGY UNCERTAINITY PRINCIPLE – DERIVATION

Consider the case of a free particle with rest mass m_0 moving

$$E = \frac{1}{2} m_0 \, v_x^2 = \frac{p_x^2}{2 m_0} \qquad \qquad \ldots(1)$$

with a velocity of V_x. The kinetic energy of the particle is given by:

If Δp_x and ΔE be the uncertainties in momentum and energy respectively, then differentiating equ (1), we have

$$\Delta E = \frac{2 \, p_x \, \Delta p_x}{2 \, m_0}$$

or

$$p_x \, \Delta p_x = m_0 \, \Delta E$$

$\therefore$

$$\Delta p_x = \frac{m_0}{p_x} \Delta E = \frac{1}{v_x} \Delta E. \qquad ...(2)$$

Further, let the uncertainty in time interval for measurement at point x be Δt, then the uncertainty Δx in position is:

$$\Delta x = v_x \, \Delta t \qquad ...(3)$$

From equs.(2) and (3), we get:

$$\Delta x \, \Delta p_x = \Delta t \cdot \Delta E. \qquad ...(4)$$

We know that

$$\Delta x \, \Delta p_x \geq \frac{h}{4 \, \pi}$$

$\therefore$

$$\Delta t \cdot \Delta E \geq \frac{h}{4 \, \pi}. \qquad ...(5)$$

Hence the Energy – Time Uncertainty relation is derived from x – p Uncertainty relation.

UNIT – III QUANTUM MECHANICS

Q.1) Write the Physical Significance of Wave function ψ

A) The Physical Significance of ψ is as follows:

1) The variable quantity which describes and also characterizes de-Broglie wave is called Wave function ψ.
2) ψ is a complex quantity that cannot be measured.
3) Schrodinger gave a simple interpretation of Wave function ψ in terms of Charge density

If ψ is the amplitude of matter waves, then ψ^2 is a measure of particle density. This when multiplied by charge of the particle, Charge density is obtained. In this way ψ^2 is a measure of Particle Charge density.

4) Max Born interpreted that $\psi \, \psi^* = |\psi|^2$ gives the probability of finding the electron in a state ψ i.e. ψ^2 is a measure of Probability density.

The probability of finding a particle in a volume $d\tau = dx \, dy \, dz$ is evaluated so that the total probability of finding the particle is given by the condition known as Normalization Condition and is represented as: $\iiint |\psi|^2 \, dx \, dy \, dz = 1$

The Wave function satisfying the above condition is called Normalized wave function.

5) The Zero probability corresponds to the certainty of not finding the particle some-where in space. $\iiint |\psi|^2 \, dx \, dy \, dz = 0$

Q.2) Explain Eigen functions and Eigen values

A) Every physical observable in Classical mechanics is expressed as an Operator in Quantum mechanics. An Operator is a entity which changes or tries to change a function into a new function. Addition, multiplication, differentiation etc all can be expressed as operators.

When any operator $\widehat{O}$ is operated on a function f(x) then it transforms it into a new function g(x) as:

$$\widehat{O}[f(x)] = \lambda [g(x)]$$

Also when any operator $\widehat{O}$ is operated on a wave function ψ then the result is given as: $\widehat{O}$ ψ = a ψ

Here the observable quantity **a** is called the **Eigen Value** of the Operator $\widehat{O}$ and ψ is called the **Eigen function** of the operator.

Examples: **Schrodinger Wave equation** in the **Operator form** can be expressed as

$$\widehat{H} \psi_n = E_n \psi_n \text{ where operator } \left[-\frac{\hbar^2}{2m} \nabla^2 + V \right] \quad \left(\because \hbar = \frac{h}{2\pi} \right)$$

Here E_n are the eigen values of $\widehat{H}$ the Hamiltonian Operator and ψ_n are corresponding eigen functions.

Q.3) Write the Postulates of Quantum Mechanics

A) Postulates of Quantum Mechanics are stated as follows:

Postulate I. Wave function is associated to describe Physical System

Every physical system is associated with a Wave function ψ and any state of system is fully described by it. Hence ψ also known as State function.

To describe a Physical System following are the boundary conditions to be satisfied:

a) Ψ(x) as well dΨ(x) must be finite,
b) Ψ(x) as well dΨ(x) /dx must be continuous and
c) Ψ(x) as well dΨ(x) /dx must be single valued
d) Normalization condition also should be satisfied

Postulate II. With every Dynamical Variable there is associated an Operator The table below shows some quantum mechanical Operators:

Classical quantity	Quantum Mechanical Operator
Cartesian components of position x, y, z	$\hat{x}, \hat{y}, \hat{z}$
Position Vector **r**	$\hat{r}$
Momentum **p**	$(-i\hbar \vec{\nabla})$
Cartesian components of linear momentum p_x, p_y, p_z	$\left(-i\hbar \frac{\partial}{\partial x}, -i\hbar \frac{\partial}{\partial y}, -i\hbar \frac{\partial}{\partial z} \right)$
Total energy E	$\left(i\hbar \frac{\partial}{\partial t} \right)$

Here the cap (^) on the symbol signifies that it is an operator.

Postulate III. Expected value of a Dynamical Quantity

The expected or average value of a dynamical quantity is the mathematical expectation of a result of single experiment

The average value or expectation value of an observable of a physical system in state ψ is given by:

$$<O> = \frac{\int \psi^* \widehat{O} \psi \, dx \, dy \, dz}{\int \psi^* \psi \, dx \, dy \, dz}$$

Q.4) Derive Schrodinger Time Independent Wave Equation

A) Consider a System of Stationary waves associated with a particle of mass m. Let the displacement of Matter waves at any instant of time t and in space with coordinates x, y, z be represented by **ψ,** known as Wave function, which is finite, continuous and periodic.

The classical differential Wave equation is given by:

$$d^2y/dt^2 = v^2 \, d^2y/dx^2$$

Which is modified to describe matter waves is given as:

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) = v^2 \, \nabla^2 \psi, \qquad \ldots(1)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad [\nabla^2 \text{ being Laplacian operator}]$$

Where and v is wave velocity

The solution of above partial differential equation (1) is given by

$$\psi = \psi_0 \sin \omega t = \psi_0 \sin 2\pi v t, \qquad \ldots(2)$$

Where v is the frequency of the stationary wave associated with the particle

Differentiating eq. (2) twice, we get

$$\frac{\partial \psi}{\partial t} = \psi_0 (2\pi v) \cos 2\pi v t$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\psi_0 (2\pi v)^2 \sin 2\pi v t$$

$$\frac{\partial^2 \psi}{\partial t^2} = -4\pi^2 v^2 \psi = -\frac{4\pi^2 v^2}{\lambda^2} \psi \qquad (\because \; v = v/\lambda) \qquad \ldots(3)$$

Substituting the value of $\left(\dfrac{\partial^2 \psi}{\partial t^2} \right)$ from eq. (3) in eq. (1), we get

$$v^2 \nabla^2 \psi = -\frac{4\pi^2 v^2}{\lambda^2} \psi$$

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0. \qquad \ldots(4)$$

Now from the de-Broglie relation, $\lambda = h/mv$

$$\therefore \qquad \nabla^2 \psi + \frac{4\pi^2}{h^2} m^2 v^2 \psi = 0. \qquad \ldots(5)$$

If E and V be the total and potential energies of the particle respectively, then its kinetic energy $\frac{1}{2} m v^2$ is given by

$$\frac{1}{2} m v^2 = E - V$$

or

$$m^2 v^2 = 2m(E - V) \qquad \ldots(6)$$

From eqs. (5) and (6), we have

$$\nabla^2 \psi + \frac{4\pi^2}{h^2} \times 2m(E - V)\psi = 0$$

or

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0 \qquad \ldots(7)$$

Eq. (7) is known as Schrödinger time-independent wave equation.

Substituting $\hbar = \dfrac{h}{2\pi}$ in eq. (7), the Schrödinger wave equation can be written as

$$\nabla^2 \psi + \frac{2m}{\hbar^2}(E - V)\psi = 0 \qquad \text{...(8)}$$

Eq. (8) can also be expressed in the following way :

$$\left(\frac{\hbar^2}{2m}\right)\nabla^2\psi + (E - V)\psi = 0$$

or

$$\frac{\hbar^2}{2m}\nabla^2\psi - V\psi = -E\psi$$

or

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = E\psi$$

or

$$\hat{H}\psi = E\psi \qquad \text{...(9)}$$

where $\hat{H} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi$, and H is known as Hamiltonian operator.

For a free particle $V = 0$, hence the Schroedinger wave equation for a free particle can be expressed as

$$\nabla^2\psi + \frac{2mE}{\hbar^2}\psi = 0 \qquad \text{...(10)}$$

Q.5) Derive Schrodinger Time Dependent Wave Equation

A) The Schrodinger Time Dependent Wave Equation can be obtained from Schrodinger Time independent Wave Equation by eliminating E.

The differential equation representing a one-dimensional wave motion is

$$\frac{\partial^2\psi}{\partial t^2} = v^2 \nabla^2\psi. \qquad \text{...(1)}$$

Considering ψ to be a complex function of space coordinates of the particle and time, the general solution of eq. (1) is given by

$$\psi(x, y, z, t) = \psi_0(x, y, z)\, e^{-i\omega t}$$

or

$$\psi = \psi_0 e^{-i\omega t} \qquad \text{...(2)}$$

Differentiating eq. (2) with respect to time t, we have

$$\frac{\partial\psi}{\partial t} = \psi_0\,(-i\omega)\,e^{-i\omega t}$$

$$= \psi_0\,(-i2\pi\nu)\,e^{-i\omega t} = -2\pi i\nu\psi$$

$$= -2\pi i(E/h)\psi \qquad (\because\ E = h\nu\ \text{ or }\ \nu = E/h)$$

$$= -\frac{iE}{\hbar}\psi \qquad (\text{where }\ \hbar = h/2\pi)$$

$$\therefore \quad E\psi = i\hbar\frac{\partial\psi}{\partial t} \qquad \text{...(3)}$$

Substituting the value of $E\psi$ in Schrödinger time-independent wave equation, we get

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left[i\hbar\frac{\partial\psi}{\partial t} - V\psi\right] = 0$$

$$\nabla^2\psi = -\frac{2m}{\hbar^2}\left[i\hbar\frac{\partial\psi}{\partial t} - V\psi\right]$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}. \qquad \text{...(4)}$$

This equation is known as Schrödinger time-dependent wave equation.

Eq. (4) can be written as

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial}{\partial t}\psi$$

or

$$\hat{H}\psi = \hat{E}\psi \qquad \qquad \dots(5)$$

where

$$\hat{H} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right) = \text{Hamiltonian operator}$$

and

$$\hat{E} = i\hbar\frac{\partial}{\partial t} = \text{Energy operator}$$

Eq. (5) describes the motion of a non-relativistic material particle.

Q.6) Derive the energy eigen values and eigen functions for a Particle in a One Dimensional Potential Well or Box (Or) Obtain an expression for the Energy of a Particle in a Box

A) Let us consider a particle of mass m moving along X-axis between two rigid walls A and B at x=0 and x=a, as shown in the fig.1. The particle moves freely between the walls. The Potential energy of the particle is constant as no force acts on it. For simplicity it is say zero.

The particle gets reflected back when it strikes the walls as they are rigid. The force and potential of the particle are related as: $|F| = \partial V/\partial x$. As $\Delta V \to \infty$ as $\Delta x \to 0$ such that $\partial V/\partial x$ has a Finite value. So the potential becomes infinite at the walls. The Potential Function is thus defined as:
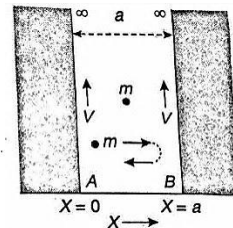
$$V(x) = \infty \quad \text{for} \quad x < 0 \quad \text{and} \quad x > a$$

and

$$V(x) = 0 \quad \text{for} \quad 0 \le x \le a \qquad \qquad \dots(1)$$

The Schrödinger wave equation for the particle is given by

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$$

As $V = 0$ between the walls, hence the equation has the following form

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = 0. \qquad \qquad \dots(2)$$

Let $\dfrac{8\pi^2 m}{h^2}E = k^2$, then eq. (2) takes the form

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0. \qquad \qquad \dots(3)$$

The solution of eq. (3) is of the form of $\sin k x$ or $\cos k x$. The general solution is given by

$$\psi(x) = A\sin k x + B\cos k x \qquad \qquad \dots(4)$$

where A and B are two constants. The values of these constants can be obtained by applying the boundary conditions of the problem. Here the particle is enclosed between two rigid walls and we know that ψ^2 represents the probability of finding the particle at any instant. The particle cannot penetrate the walls, hence $\psi = 0$ at $x = 0$ and $\psi = 0$ at $x = a$. These are the boundary conditions. Applying these conditions, we have

$$0 = A\sin 0 + B\cos 0, \quad i.e., B = 0$$

$$\psi(x) = A\sin k x$$

Again
$$0 = A \sin k a \quad \therefore \quad \psi(x) = 0 \quad \text{when } x = a$$
Here either $A = 0$ or $\sin k a = 0$ but $A \neq 0$ because if $A = 0$, the entire function will be zero as $B = 0$

$\therefore$
$$\sin k a = 0 \quad \text{or} \quad k a = n \pi (n = 0, 1, 2, 3, \ldots)$$

or
$$k = \frac{n \pi}{a} \qquad \ldots(5)$$

Now the wave function becomes

$$\psi(x) = A \sin \frac{n \pi x}{a} \qquad \ldots(6)$$

Eigen Values

From eq. (5), $k^2 = \frac{n^2 \pi^2}{a^2}$ and we have used k^2 for $\frac{8\pi^2 m E}{h^2}$

$\therefore$
$$\frac{8 \pi^2 m E_n}{h^2} = \frac{n^2 \pi^2}{a^2} \qquad \text{(using } E_n \text{ for } E, \text{ in general.)}$$

or
$$E_n = \frac{n^2 h^2}{8 m a^2} = n^2 \frac{\pi^2 \hbar^2}{2 m a^2} \qquad \ldots(7)$$

$$(n = 1, 2, 3, \ldots) \quad \text{and} \quad (\hbar = h/2\pi)$$

It is clear from expression (7) that inside an infinitely deep potential well, the particle can have only discrete set of values of energy, i.e., the energy of the particle is quantized. The discrete energy values are given by :

$$E_1 = \frac{\pi^2 \hbar^2}{2 m a^2} \text{ for } n = 1, E_2 = 4 \frac{\pi^2 \hbar^2}{2 m a^2} = 4 E_1, \text{ for } n = 2$$

$$E_3 = 9 \frac{\pi^2 \hbar^2}{2 m a^2} = 9 E_1 \text{ for } n = 3, E_4 = 16 \frac{\pi^2 \hbar^2}{2 m a^2} = 16 E_1 \qquad \text{for } n = 4 \text{ and so on.}$$
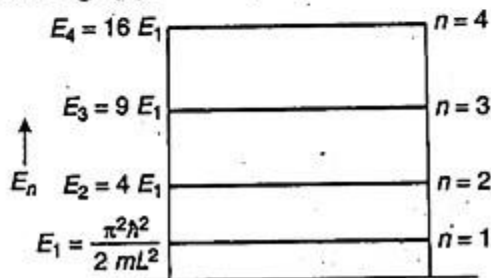
The energy levels are shown in fig. (2).

Fig. (2)   Energy level of a particle inside an infinite potential well

Eigen Functions

The constant A of eq. (6) can be obtained by applying this normalization condition, i.e.,

$$\int_0^a |\psi(x)|^2 \, dx = 1 \quad \text{or} \quad \int_0^a A^2 \sin^2 \frac{n \pi x}{a} \, dx = 1$$

or
$$A^2 \int_0^a \sin^2 \frac{n \pi x}{a} \, dx = 1$$

or
$$A^2 \int_0^a \frac{1}{2}\left[1 - \cos \frac{2 n \pi x}{a}\right] dx = 1$$

or
$$\frac{A^2}{2}\left[x - \frac{a}{2\pi n}\sin\frac{2n\pi x}{a}\right]_0^a = 1$$

or
$$\frac{A^2 a}{2} = 1 \quad \text{or} \quad A^2 = \frac{2}{a} \quad \text{or} \quad A = \sqrt{\left(\frac{2}{a}\right)}$$

$\therefore$
$$\psi(x) = \sqrt{\left(\frac{2}{a}\right)}\sin\frac{n\pi x}{a}$$
...(8)

Eq. (8) gives the wave functions of the particle enclosed in infinitely deep potential well. The wave functions ψ_1, ψ_2 and ψ_3 corresponding to $n = 1, 2$ and 3 are shown in fig. (3).
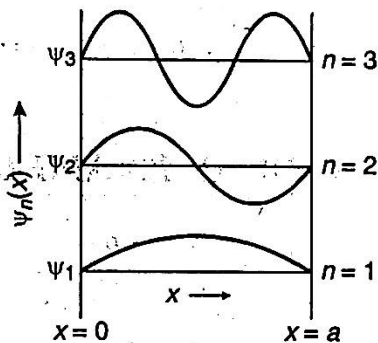

Fig. (3) Wave functions of a particle enclosed in an infinite potential well.

UNIT – IV 4 GENERAL PROPERTIES OF NUCLEI

Q.1) Explain various Properties of Nucleus

A) Some of the important properties of atomic Nucleus are:

   a) **Nuclear Mass:** This is the sum of the masses of protons and neutrons contained in it, is expressed in terms of atomic mass unit (a.m.u). 1 a.m.u = 1.66×10^{-27} kg = 931.48 MeV. For example, $^{12}_{6}C$ nucleus has a mass of 12 a.m.u

   b) **Nuclear Binding Energy:** Nuclear binding energy is the energy required to separate an atomic nucleus completely into its constituent protons and neutrons, or, the energy that will be released by combining individual protons and neutrons into a nucleus.

Mass of the Nucleus should be $Zm_p + Nm_n$. But it was found that the real nuclear mass is $< Zm_p + Nm_n$. This difference in mass is known as Mass Defect. If M is the real mass of nucleus, then Mass Defect $\Delta M = Zm_p + Nm_n - M$. Binding Energy of a nucleus is

$$B.E = \Delta M.C^2 = (Zm_p + Nm_n - M) C^2$$

   c) **Nuclear Charge:** The charge on the nucleus is due to the protons contained in it. The total charge of any nucleus is equal to the charge of all protons inside in it. It is given by $+Ze$ where Z is the atomic number of the nucleus and e is the charge of each proton = 1.6×10^{-19}C.

   d) **Nuclear Radius (Size):** Most of the nuclei being spherical, It is observed that the radius of them is approximately directly proportional to their mass number A. The distance of closest approach of α-particle to nucleus is regarded as the measure of nuclear size or the radius.

It is given by the empirical relation: $r \propto A^{1/3}$ or $r = r_0 A^{1/3}$, where is a constant with average value=1.4×10^{-15}m. example Carbon $(A = 12)$

$$r = 1.4 \times 10^{-15} \times (12)^{1/3} = 3.21 \times 10^{-15} \text{ m}$$

   e) **Nuclear Density:**

The density of the nucleus can be calculated as follows :

$$\text{Volume of the nucleus} = \frac{4}{3}\pi r^3,$$

where r is the radius of the nucleus.

Taking $r = 1.5 \times 10^{-15} A^{1/3}$ metre, we have

$$\text{Volume of the nucleus} = \frac{4}{3}\pi (1.5 \times 10^{-15})^3 A \text{ metre}^3 = 14.15 \times 10^{-45} A \text{ metre}^3$$

$$\text{Mass of the nucleus} = A \times \text{mass of proton (approximately)} = 1.673 \times 10^{-27} A \text{ kg}$$

$$\therefore \quad \text{Density of nucleus} = \frac{1.673 \times 10^{-27} A}{14.15 \times 10^{-45} A} \text{ kg/metre}^3$$

$$= 1.18 \times 10^{17} \text{ kg/metre}^3$$

   f) **Nuclear Quantum States:** From the artificial radioactivity and α, γ – ray spectra, it was observed that all nuclei possess a set of discrete energy levels i.e. quantized energy levels.

   g) **Nuclear Spin (or) Total Angular Momentum:** Both proton and neutron inside the nucleus have an intrinsic angular momentum, referred to as Spin, whose magnitude is similar to electron, given by $\frac{1}{2}\hbar$. In addition to this, nucleons also possess orbital

angular momentum due to their motion about center of nucleus. Hence there arises a total angular momentum of Nucleus **I** referred to as the nuclear spin, whose magnitude is given by: $\hbar[I\,(I+1)]^{1/2}$ where $\hbar$... I value depends upon type of nucleon interaction.

h) **Magnetic Dipole Moment of Nuclei:** Charged particle moving in closed path produces a magnetic field. At large distance, this is due to a magnetic dipole located at the current loop. Spinning electron is associated with a magnetic dipole moment of 1 Bohr magneton
i.e. $\mu_e = \dfrac{e\hbar}{2m_e}$. Here e is charge & m_e is mass of electron
According to Dirac's theory, the spinning proton has a magnetic dipole moment given by: $\mu_N = \dfrac{e\,(\hbar/2\pi)}{2m_p} = \dfrac{e\hbar}{2m_p}$

Where m_p is the mass of proton and μ_N is called Nuclear magneton. It was found that even a neutron possesses a dipole moment, $\mu_n = -1.9128\,\mu_N$ basing on meson theory.

i) **Electric Quadrupole moment (Q):** This is due to the consequence of symmetry of nucleus about the center of mass. The deviation from the spherical symmetry about center of mass is expressed in terms of a quantity known as electric quadrupole moment.Considering the symmetry as an ellipsoid of revolution, let the diameter along axis is 2a and diameter in perpendicular direction be 2b, as shown in the figure:

$Q = 0$ $Q = -ve$ $Q = +ve$
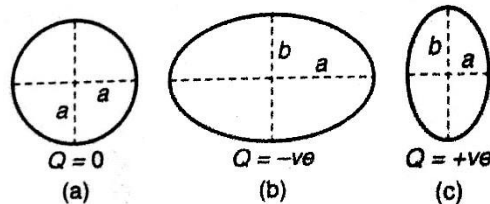(a)   (b)   (c)

Fig: Shape of Nucleus and Electric quadrupole moments.

It is given by the equation: $Q = \dfrac{2}{5}Ze[b^2 - a^2]$ where Ze is the total charge on nucleus and Z is the atomic number i.e. the number of protons. It is clear from the expression that:

(i) $Q = 0$ for spherical shaped nucleus

(ii) $Q = $ -ive, for ellipsoid (a>b) i.e. oblate spheroid

(iii) $Q = $ +ive, for ellipsoid (b>a) i.e. prolate spheroid It was found that $Q = 0$ for those nuclei whose spin quantum number is I=0 or $\frac{1}{2}$.

Such nuclei have spherical charge distribution.

UNIT – IV 5 RADIOACTIVITY & DECAY

Q.2) Explain the Liquid drop model of Nucleus

A) Some of the important postulates of Liquid drop model of Nucleus are:

1) This model was proposed by Neil's Bohr
2) According to this model, the Nucleus is similar to a small electrically charged liquid drop i.e. the nucleus takes spherical shape for its stability
3) The nucleons move within this spherical enclosure like molecules in a liquid drop

4) The motion of the nucleons within the nucleus is a measure of nuclear temperature as the molecular motion of molecules in a liquid is the measure of its temperature
5) The nucleons always remain a constant apart and share among them the total energy of the nucleus

6) The nucleons deep inside the nucleus are attracted from all sides by the neighbouring nucleons while those on the surface are attracted from one side only. Hence in this way the B.E for surface nucleons is smaller than the binding energy for inner nucleons Following are the analogies or similarities between Liquid Drop and Nucleus:

1) Both are spherical in nature

2) In both of them density is independent of volume with exceptions that: density of nucleus is independent of nature while density of liquid drop depends on its type

3) Both molecules of liquid drop and nucleons in nucleus interact over short ranges

4) As the surface tension acts on the surface of a liquid drop, a potential barrier acts on the surface of the nucleus

5) When the temperature of the molecules in a liquid drop increased, vaporization of molecules takes place. When the nucleons in the nucleus are subjected to external energy, a compound nucleus is formed, which emits nucleons almost immediately. This process is known as Nuclear Fission. Hence vaporization is similar to nuclear fission

Merits:

a) It is successfully applied in describing Nuclear Reactions and explaining Nuclear Fission

b) Calculation of Atomic Masses and Binding Energies can be done with great accuracy
   Demerits:

a) Failed to explain the Nuclear Stability

b) Failed to explain Magic numbers, their exceptional stability

Q.3) Explain the Shell model of Nucleus

A) Some of the important postulates of Shell model of Nucleus are:

1) According to Shell model, the nucleus consists of a series of protons and neutrons placed in a certain discrete energy levels or shells just like the electrons in the discrete shells of atom

2) According to the Pauli's exclusion principle, two protons with opposite spins and two neutrons having opposite spins are accommodated in a particular shell. In this way the first shell consists of 2 protons & 2 neutrons more tightly bound than other shells.

The concept of Shell model is based on the following experimental facts & observations:

1) The nuclei for which Z (No. of Protons) and A-Z (No. of Neutrons) are the following:
   2 $[{}_{2}^{4}He, Z = 2 \; \& \; A - Z = 4]$; 8 $[{}_{8}^{16}O, Z = 8 \; \& \; A - Z = 8]$ are more stable than their neighbors

2) The nuclei for which Z or A-Z is Magic number are specially stable. For example ${}_{28}^{62}Ni \; [Z = 28 \; \& \; A - Z = 34]$. The binding energy curve shows breaks or kinks at these nuclei which correspond to sudden increase in their binding energies per nucleon

3) The electric quadrupole moments of magic number nuclei are very low compared with those other nuclei. This is expected for more stable nuclei

4) Each nucleon moves freely in a force field described by the potential which is a function of radial distance from the center of the nucleus

5) The energy levels (or) shells are filled according to Pauli's exclusion principle
6) Explanation of comparative study of even-even & odd-odd nuclei: It has been observed that even-even nuclei are in general more stable than odd-odd nuclei. According to the Pauli's principle, single sublevels can have a maximum of two nucleons, one has spin up and other spin down. In this way, an even-even nucleus contains completely filled shell and hence shows a greater stability. On the other hand, odd-odd nucleus contains incompleted subshells and hence lower stability
7) Prediction of angular momentum of nuclei: It has been observed that in even-even nuclei, all protons and neutrons pair up to cancel out one another spin and orbital angular momenta. So the total angular momentum is zero for such nuclei. Now for the case of odd-odd nuclei, there exists an extra proton and extra neutron with half integral spins each giving rise to total integral angular momentum

Merits: 1) This model successfully accounted for the Magic numbers
2) Also explained Nuclear stability, Observed Angular momenta, Magnetic moments and electric quadrupole moments of nuclei

Demerits: This model fails to explain and account for large quadrupole moments and spherical shapes of many nuclei **Q.4) What are Magic numbers?**

A) The Magic number is a number of nucleons (either protons or neutrons or both) such that they are arrangement into complete shells within atomic nucleus

The seven most recognized magic numbers are: 2, 8, 20, 28, 50, 82 and 126

The property of these magic number nuclei is that nuclei consisting of such magic number of Nucleons have exceptional stability due to higher binding energy per nucleon

Q.5) Explain Gamow's theory of α-Decay

A) Classically it was assumed that an α-particle can come out of $^{238}_{92}U$ if its energy is more than 35.6 MeV. But experimentally it was found that α-particle comes out of $^{238}_{92}U$ with a K.E about 4MeV.

Hence Classical theory fails to explain α-decay as it cannot escape. George Gamow showed that Quantum mechanically there is finite probability for α-particle to penetrate the potential barrier and escape the nucleus. Gamow made the following assumptions:
1) The α-particle will not pre-exist inside the nucleus rather it is formed during the disintegration process
2) The α-particle will be in constant motion, bouncing back & forth between walls of potential well. During each collision with the wall, there is a definite probability of leakage through the potential barrier
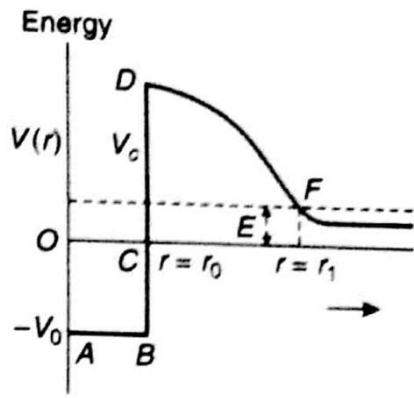
Let P be the probability of escape of α-particle in each collision. Let ν be the frequency with which the α-particle collides with walls to escape. Now the decay probability per unit time is given by:
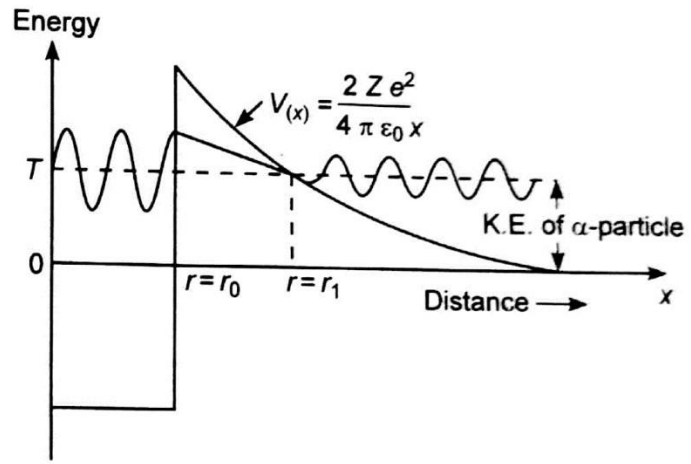$$\lambda = \nu\,P \text{ -----------> (1)}$$

Where frequency of α-particle will be given by $\nu = \dfrac{V}{2r_0}$ -----------> (2),

V is the velocity & $2r_0$ is the radial distance

Hence from (1) & (2) equations, $\lambda = \dfrac{V}{2r_0}\,P$ -----------> (3)

(a)   (b)

Fig. (a) represents approximate form of Potential energy of α-particle as a function of its distance
In the Fig.(a), $OA = -V_0$ is the attractive potential when α-particle is inside potential well; $CD = V_c$ the
repulsive columbic potential when outside the well i.e. into the barrier. Fig.(b) represents the
escaping of α-particle from barrier quantum mechanically i.e. as an α-de Broglie Wave with a finite
probability.

Using the W.K.B approximation, the Probability P can be calculated from the formula:

$$\log_e P = -\frac{2}{\hbar} \int_{r_0}^{r_1} \sqrt{\{2\, m V(r) - E\}}\ dr \qquad ...(4)$$

where m is the mass of the particle and $\hbar = h/2\pi$.

Further, $$V(r) = \frac{2\, Z\, e^2}{4\, \pi \varepsilon_0\, r} \qquad ...(5)$$

is the electrostatic potenital energy of α-particle at a distance r from the nucleus of charge $Z\, e$. The charge
on α-particle is $2\, e$.

∴ $$\log_e P = -\frac{2}{\hbar} \int_{r_0}^{r_1} \sqrt{\left\{2\, m \left(\frac{2\, Z\, e^2}{4\, \pi \varepsilon_0\, r} - E\right)\right\}}\ dr \qquad ...(6)$$

At $r = r_1, E = 2\, Z\, e^2 / (4\, \pi \varepsilon_0\, r_1)$

∴ $$\log_e P = -\frac{2}{\hbar} \int_{r_0}^{r_1} \sqrt{\left\{2\, m \left(\frac{E\, r_1}{r} - E\right)\right\}}\ dr$$

or $$\log_e P = -\frac{2}{\hbar} (2\, mE)^{1/2} \int_{r_0}^{r_1} \left(\frac{r_1}{r} - 1\right)^{1/2} dr \qquad ...(7)$$

Let us solve the integral $$\int_{r_0}^{r_1} \left(\frac{r_1}{r} - 1\right)^{1/2} dr.$$

Let us substitute $r = r_1 \cos^2 \theta$ and $r_0 = r_1 \cos^2 \theta_0$

$$\therefore \quad dr = -2r_1 \cos\theta \sin\theta \, d\theta = \int_{\theta_0}^{0} \left(\frac{r_1}{r_1 \cos^2\theta} - 1\right)^{1/2} (-2r_1 \cos\theta \sin\theta \, d\theta)$$

$$= -2r_1 \int_{\theta_0}^{0} (1 - \cos^2\theta)^{1/2} \sin\theta \, d\theta = -2r_1 \int_{\theta_0}^{0} \sin^2\theta \, d\theta$$

$$= -2r_1 \int_{\theta_0}^{0} \frac{1}{2}(1 - \cos 2\theta) \, d\theta = -2r_1 \left[\frac{1}{2}\theta - \frac{1}{2}\frac{\sin 2\theta}{2}\right]_{\theta_0}^{\theta}$$

$$= -2r_1 \left[-\frac{1}{2}\theta_0 + \frac{1}{4}\sin 2\theta_0\right] = r_1 [\theta_0 - \sin\theta_0 \cos\theta_0]$$

As $\qquad \cos^2\theta_0 = \dfrac{r_0}{r_1}$ hence $\quad \theta_0 = \cos^{-1}\sqrt{\left(\dfrac{r_0}{r_1}\right)}$ and $\quad \sin\theta_0 = \sqrt{\left(1 - \dfrac{r_0}{r_1}\right)}$

$$\therefore \qquad \int_{r_0}^{r_1} \left(\frac{r_1}{r} - 1\right)^{1/2} dr = r_1 \left[\cos^{-1}\sqrt{\left(\frac{r_0}{r_1}\right)} - \sqrt{\left(1 - \frac{r_0}{r_1}\right)}\sqrt{\left(\frac{r_0}{r_1}\right)}\right] \qquad \ldots(8)$$

From equations (7) and (8), we get

$$\log_e P = -\frac{2}{\hbar}(2mE)^{1/2} r_1 \left[\cos^{-1}\sqrt{\left(\frac{r_0}{r_1}\right)} - \sqrt{\left(1 - \frac{r_0}{r_1}\right)}\sqrt{\left(\frac{r_0}{r_1}\right)}\right] \qquad \ldots(9)$$

Because the potential barrier is relatively wide ($r > r_0$), hence

$$\cos^{-1}\left(\frac{r_0}{r_1}\right)^{1/2} \approx \frac{\pi}{2} - \left(\frac{r_0}{r_1}\right)^{1/2} \quad \text{and} \quad \left(1 - \frac{r_0}{r_1}\right)^{1/2} \approx 1.$$

Substituting these values in eq. (9), we get

$$\log_e P = -\frac{2}{\hbar}(2mE)^{1/2} r_1 \left[\frac{\pi}{2} - \left(\frac{r_0}{r_1}\right)^{1/2} - \left(\frac{r_0}{r_1}\right)^{1/2}\right]$$

$$= -\frac{2}{\hbar}(2mE)^{1/2} r_1 \left\{\frac{\pi}{2} - 2\left(\frac{r_0}{r_1}\right)^{1/2}\right\}$$

Here, $\qquad r_1 = \dfrac{2Ze^2}{4\pi\varepsilon_0 E}$

$$\therefore \qquad \log_e P = -\frac{2}{\hbar}(2mE)^{1/2} \frac{2Ze^2}{4\pi\varepsilon_0 E}\left[\frac{\pi}{2} - 2\times\left(\frac{4\pi\varepsilon_0 E r_0}{2Ze^2}\right)^{1/2}\right]$$

or $\qquad \log_e P = \dfrac{4e}{\hbar}\left(\dfrac{m}{\pi\varepsilon_0}\right)^{1/2} Z^{1/2} r_0^{1/2} - \dfrac{e^2}{\hbar\varepsilon_0}\left(\dfrac{m}{2}\right)^{1/2} Z E^{-1/2}. \qquad \ldots(10)$

Substituting the values of various constants in eq. (10), we have

$$\log_e P = 2.97 \, Z^{1/2} \, r_0^{1/2} - 3.95 \, Z \, E^{-1/2}. \qquad \ldots(11)$$

Here E = kinetic energy in MeV, r_0 = nuclear radius in fermi (1 fermi $= 10^{-15}$ m) and Z = atomic number of nucleus − alpha particle.

The decay constant is given by

$$\lambda = vP = \left(\frac{v}{2r_0}\right)P$$

$\therefore$
$$\log_e \lambda = \log_e \left(\frac{v}{2r_0}\right) + \log_e P$$

$$= \log_e \left(\frac{v}{2r_0}\right) + 2.97 \, Z^{1/2} \, r_0^{1/2} - 3.95 \, Z \, E^{-1/2} \qquad \ldots(12)$$

Eq. (12) represents the α-decay.

When base of $\log_e$ is changed to base$_{10}$, we have

$$\log_{10} \lambda = \log_{10} \left(\frac{v}{2r_0}\right) + 1.29 \, Z^{1/2} \, r_0^{1/2} - 1.72 \, Z \, E^{-1/2}. \qquad \ldots(13)$$

Geiger and Nuttal measured the range of α-particles emitted by several radioactive elements and found that there exists a regular relationship between the ranges and the half lives of the elements. The relationship is expressed as

$$\log \lambda = A + B \log R$$

where R is the range of α-particles emitted by an element, λ is its disintegration constant and A and B are constants. This relation is called Geiger-Nuttal law.

The law is helpful in determining roughly the decay constants of readioactive substances of very short or very long lives.

If a graph is plotted between logarithms of distintegration constant λ and the logarithms of range R for various radioactive substances emitting α-particles in the three series, nearly parallel straight lines are obtained as shown in fig. (2). It is obvious from the figure that the three lines have different intercepts on $\log R$ axis. This indicates that A is different for different series while B is same for all series.
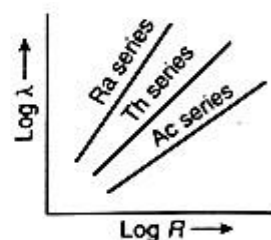

Fig.

BETA Decay

β-decay is the process of emission of electron (beta radiation) from the nucleus. There are three modes of radioactivity connected with β-decay. They are :

1. *The emission of electron* (β^-)
2. *The positron emission* (β^+)
3. *Electron capture* (The nucleus captures one of the orbiting electron.

1. The Emission of Electron (β^-)

When a radioactive element emits a β^--particle, the product has the same mass number as the parent but the atomic number is greater by one. The process can be written as

$$_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0 \quad (\beta^{-1} \text{ decay})$$
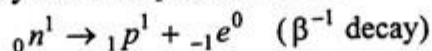
2. The Positron Emission

In positron emission from a radiactive substance, the maass number still unchanged but the atomic number of product is now one unit less than that of parent, the process can be written as

$$_1p^1 \rightarrow {}_0n^1 + {}_{+1}e^0 \quad (\beta^+ \text{ decay})$$
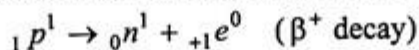
3. Electron Capture or K-electron capture

In case of a ratioactive substance having high Z value (proton number), there is another mechanism of transforming the proton into neutron.

The proton turns into a neutron by absorbing an orbiting electron instead of emitting a positron. The orbiting electron is usually captured from K-shell which is closest to the nucleus of present nuclide. So, the mechanism is known as *electron capture*. The electron interacts with a proton in the nucleus and converts it into a neutron *i.e.*, K electron falls into the nucleus. The process can be represented as

$$_{+}p^1 + _{-1}e^0 \rightarrow {}_0n^1 \qquad \text{(Electron capture)}$$

It is important to mention that the vacancy in K-shell of daughter nuclide resulting from K-capture is filled by the rearrangement of the $Z-1$ electrons in K-shell. This leads to the emission of the characteristic X-rays of the daughter element.

Q.6) Explain β-Energy Spectrum

A) β-ray Continuous & Discrete Energy Spectrum

A graph is plotted betweeen the no. of β-particles and their energy. The graph is an extensive continuum upon which a few sharp lines are superposed

The graph shows that the β-particles emitted by radioactive substance possess a continuous range of energies when G-M Counter is being used. Instead of this, when a photographic plate is used, a dark background with several dark lines is observed. This shows a   discrete behaviour.
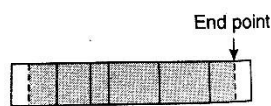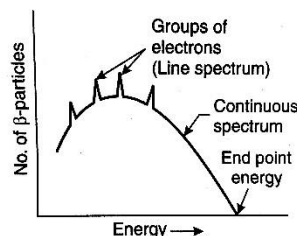

Fig. (6) β-ray spectrum

Hence the following are the characteristics of β-ray Spectrum:

1) Continuous Spectrum: β-particles have a continuous distribution of energy. The continuous spectrum will have energy ranging from 0 to a certain Maximum value.
2) Discrete Spectrum: Some of the radioactive substances emit β-particles which produce line spectra superimposed on the faint background. These are called Secondary spectra.

The Continuous spectrum is due to the electrons emitted by disintegration inside the nucleus. The Line or Discrete spectrum is due to external electrons emitted by internal conversion.

a) Primary (or) Disintegration electrons: These are emitted by nucleus with varying velocities, giving rise to Continuous spectra with an end-point or cut-off point at high energy end.
b) Secondary (or) Internal Conversion electrons: The discrete spectrum is due to these electrons arise from the process of internal conversion. The γ-rays during their escape from the nucleus knock-out the electrons from K, L, M and N shells of atom. These are the orbital or secondary electrons which produce line or discrete spectrum.

Q.7) Explain Pauli's Neutrino's Hypothesis or Write a short note on Neutrino Hypothesis

In order to explain the difficulties with β-ray spectra, Pauli in 1930 forwarded a new hypothesis known as Neutrino hypothesis. According to this hypothesis, in the process of β-decay, a neutral particle which has negligible mass as compared to electron, charge zero and spin half is emitted along with β-particles. This particle carries a part of available ciergy and momentum and is called *neutrino*. Neutrino travels with the velocity of light. In fact, in some respect, it resembles with photon.

On the basis of neutrino theory, the conservation of linear and angular momenta in β-decay may be explained.

We know that an atomic nucleus contains positively charged protons and uncharged neutrons. During β-decay, β-particles (electrons) are created just at the time of emission by the conversion of a neutron into a proton. When a neutron is converted into proton, an electron and neutrino are emitted. The β-decay can be represented by the following equation :

$$\text{Neutron} \rightarrow \text{proton} + \text{electron} + \text{neutrino}$$

$$_0n^1 \rightarrow {}_1H^1 + {}_{-1}e^0 + \nu$$

mass	$1 \rightarrow 1 + 0 + 0$
charge	$0 \rightarrow +1 + -1 + 0$

So both the mass and charge are conserved.

The neutrino theory successfully explains the continuous energy spectrum of β-rays. In β-decay the proton remains in the nucleus while both β-particle and the neutrino come out. The both particles are emitted simultaneously. The total energy of these two particles is constant and is equal to the end point energy observed in β-ray spectrum (see fig. 6). The energy is shared between β-particle and neutrino.

The problem of conservation of angular momentum is also solved by neutrino hypothesis.

UNIT – V 6 CRYSTAL STRUCTURE

Q.1) Explain fourteen Bravais lattices with neat diagrams (or) How are crystals classified? Explain with neat diagrams all Bravais lattices.

A) Bravais showed that there are only fourteen ways of arranging points in space so that the surrounding (environment) looks the same from each point. These lattices are known as Bravais lattices. These are shown in the tabular form below:

Crystal Type	Number of Possible Lattice Types
Cubic	3 – Simple/Primitive, Body Centered & Face Centered
Tetragonal	2 – Simple/Primitive, Body Centered
Orthorhombic	4 – Simple/Primitive, Body Centered, Face Centered, Base Centered
Monoclinic	2 – Simple/Primitive & Face Centered
Triclinic	1 – Simple/Primitive
Trigonal (Rhombohedral)	1 – Simple/Primitive
Hexagonal	1 – Simple/Primitive

1. Cubic lattice

It has 3 possible arrangements

(i) Simple or Primitive Cubic lattice:
There is one lattice point at each of eight corners of the unit cell.

(ii) Face Centered Cubic lattice:
There are 8 lattice points at 8 corners of the cube and 6 lattice points at the six centers of the cube

(iii) Body Centered Cubic lattice:
There are 8 lattice points at 8 corners of the cube along with one lattice point at the body center.

(a) Simple cubic   (b) Face centred cubic   (c) Body centred cubic
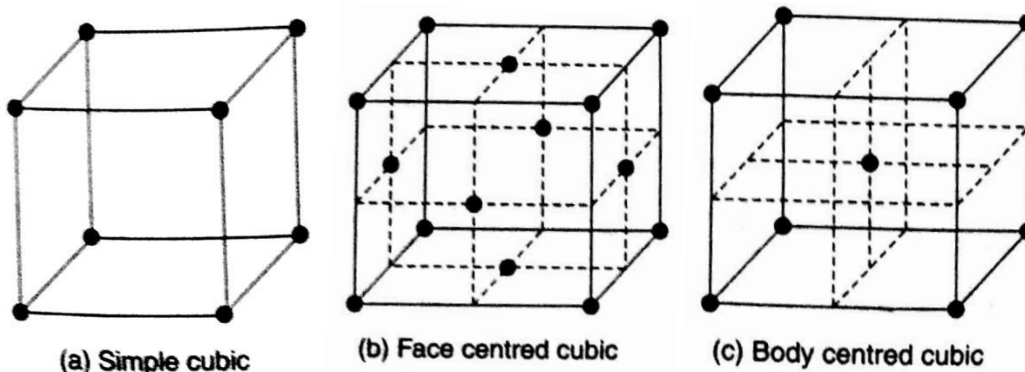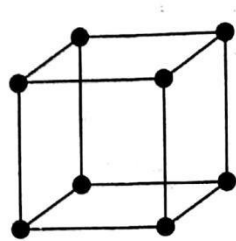
2. Tetragonal lattice

It has 2 possible arrangements

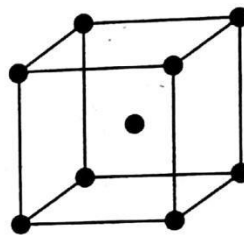**(i) Simple or Primitive tetragonal lattice:**
There is one lattice point at each of eight corners of the unit cell.

(ii) Body Centered tetragonal lattice:
There are 8 lattice points at 8 corners of the unit cell along with one lattice point at the body center.

(a) Simple   (b) Body-centred

3. Orthorhombic lattice

It has 4 possible arrangements

(i)   **Simple or Primitive Orthorhombic lattice**:

There is one lattice point at each of eight corners of the unit cell.
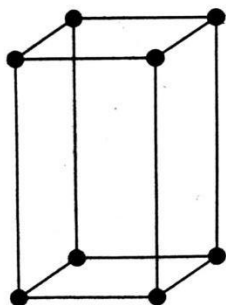
(ii) **Body Centered Orthorhombic lattice:**

There are 8 lattice points at 8 corners of the unit cell along with one lattice point at the body center.
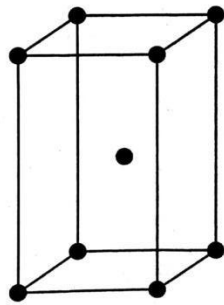
(iii) **Face Centered Orthorhombic lattice:**

There are 8 lattice points at 8 corners of the unit cell along with six lattice points at six face centers.

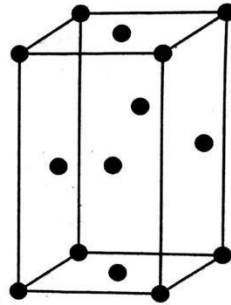**(iv)** **Base Centered Orthorhombic lattice:**

There are 8 lattice points at 8 corners of the unit cell along with two lattice points at the center of two opposite faces.
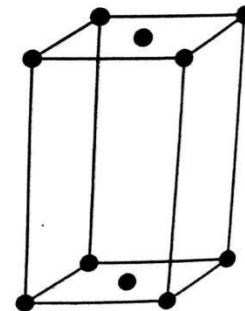

(a) Simple   (b) Body-centred   (c) Face-centred   (c) Base-centred
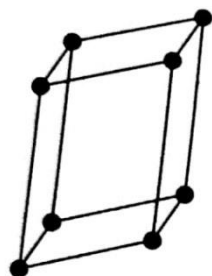
4. Monoclinic lattice

It has 2 possible arrangements or space lattices
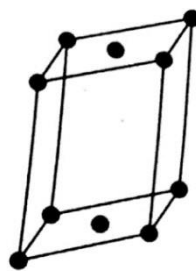
(i)   **Simple or Primitive Monoclinic lattice**:

There is one lattice point at each of eight corners of the unit cell.

(ii) **Base Centered Monoclinic lattice:**

There are 8 lattice points at 8 corners of the unit cell along with two lattice points at the center of two opposite faces.
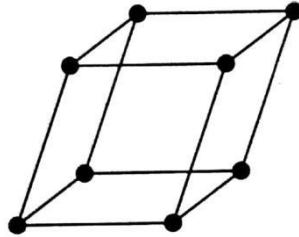

(a) Simple   (b) Base-centered

5. **Triclinic lattice**

It has only one possible arrangement or space lattice **(i)**

Simple or Primitive Triclinic lattice:

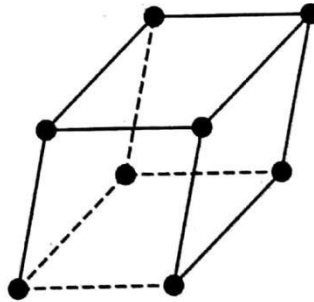There is one lattice point at each of eight corners of the unit cell.

6. **Trigonal (or) Rhombohedral lattice**

It has only one possible arrangement or space lattice

(ii)   Simple or Primitive Rhombohedral lattice:

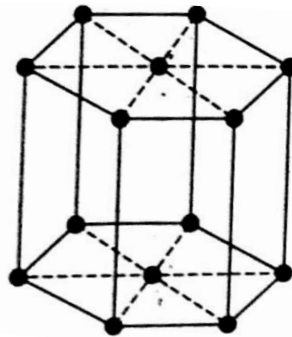There is one lattice point at each of eight corners of the unit cell.

7. **Hexagonal lattice**

It has only one possible arrangement or space lattice **(iii)**

Simple or Primitive Hexagonal lattice:

It has lattice points at all 12 corners of the hexagonal unit cell.

Q.2) What are Miller Indices? How are they calculated? (or) What are Miller Indices? How they are determined for a crystal plane?

A)   Miller Indices: Miller Indices are the three smallest possible integers which have the same ratios of the reciprocals of the intercepts of the plane concerned on the three axes.

According to Miller, the crystal planes could be described in terms of intercepts along the three axes as shown in the figure below. The intercepts on the three axes are (2,3,4). The reciprocals of these intercepts are small whole numbers. These numbers are denoted by h, k and l. These numbers are known as Miller Indices.

Miller Indices of a plane are the reciprocals of the intercepts which a plane makes with the axes when reduced to smallest numbers.

Rules for finding Miller Indices:

(i) First of all, determine the intercepts of the plane on the three coordinate axes.

(ii) Secondly, take the reciprocals of these intercepts.

(iii) Lastly, reduce the reciprocals into whole numbers. This can be done by multiplying each reciprocal by a number obtained after taking L.C.M of the denominator.

The general procedure is as follows:

(i) Let the three intercepts made by the lattice plane on the three axes (x, y, z) respectively be pa, qb and rc, where a, b, c are the lattice vectors/constants along the 3 axes and p, q, r may be either small integers or fractions.

(ii) Now take the reciprocals of p, q, r i.e. $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$

(iii) Now determine the smallest possible integers h, k, l in such a way that

$$h:k:l = \frac{1}{p}, \frac{1}{q}, \frac{1}{r}$$

The Numbers h, k, l are called the Miller Indices of the given set of planes. The plane is designated/indicated as (h k l)

Example: Let us consider the Miller Indices in the particular case of the plane that cuts the intercepts 2, 3 and 4 units along the three axes.

(i) Intercepts are: 2, 3, 4

(ii) Reciprocals of these are: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

(iii) L.C.M of denominators i.e. 2, 3, 4 is 12. Upon multiplying by 12, we have 6, 4, 3

Thus the Miller Indices of the plane is (6 4 3).

Q.2) Explain Laue's method of X-ray diffraction for the determination of Crystal structure.

A) The experimental study of large single crystals or macro crystals is due to the method conceived by Max Von Laue of Germany. The experimental arrangement of this method consists of the following apparatus:

Apparatus: source of Monochromatic X-rays, pin hole, lead diaphragms, crystal holder, photographic plate etc

Description: The experimental arrangement of the Laue method is shown in the figure

below:
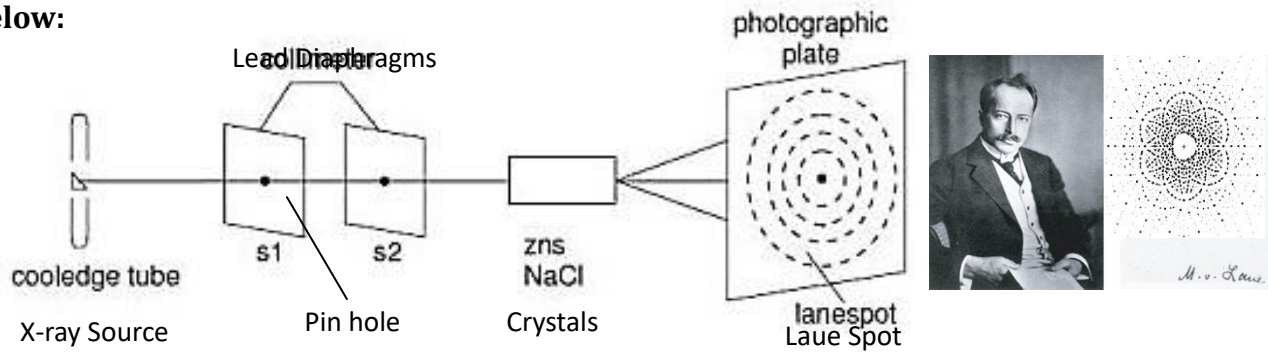
At first the Crystal is held stationary in a crystal holder. A beam of monochromatic X-rays is incident on the crystal. The X-rays after passing through the crystal are diffracted and are recorded on a photographic plate. The X-rays before passing through the crystal are collimated or limited to a fine pencil beam by a slit system i.e. through the pin hole of lead diaphragms. Here the diameter is chosen so small that a sharper interference phenomenon should occur. Some of the X-rays after penetrating through the crystal are scattered from different atomic diffraction centers because a whole range of wavelengths are present in the continuous spectrum. There will be discrete values of wave length, λ which satisfy the Bragg's condition.
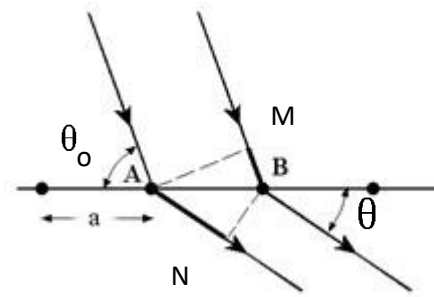
The diffraction of X-rays will occur from many families of planes at once. Each of the family picks up one of the wavelength that it diffracts at a certain angle. The diffraction pattern consists of a bright central spot and a set of spots arranged in a definite pattern about the central spot. This symmetrical pattern of spots is known as Laue Diffraction pattern.

Theory: Imagine a line grating (One Dimensional Lattice) with atoms on the line as diffraction centers. Let 'a' be the lattice constant i.e. the constant distance between two successive lattice points. Suppose monochromatic X-rays of wavelength λ are incident on the space lattice. The electrons in the atoms of the lattice points scatter X-rays in all directions coherently.

Let be the complementary angle of incidence and θ be the complementary angle of diffraction. Let AM and BN are the perpendiculars dropped on the incident and diffracted rays respectively. The diffracted rays will have maximum intensity if:

$$AN - BM = n_1 \lambda \qquad \text{------------ (1)}$$

is an integer. From the Figure:

$$AN = AB \, Cos \, \theta = a \, Cos \, \theta$$
$$BM = AB \, Cos \, \theta_o = a \, Cos \, \theta_o$$
$$\therefore \quad \text{From (1),} \quad a \, Cos \, \theta - a \, Cos \, \theta_o = \lambda$$

It is usual to denote the term a $Cos \, \theta$ **and** $Cos \, \theta_o$ **by** α **and** α_o **known as the directional**

Where

cosines of the incident and diffracted x-rays. Hence we have:

a (α - α_o) = $n_1 \lambda$

The above deduction is extended to entire crystal by considering it as a three dimensional diffraction grating. If α_o **,** β_o **,** γ_o **are direction cosines along the 3 mutually perpendicular axes and after the diffraction these are** α **,** β **,** γ **then the conditions satisfied for diffraction**

are

$$a (\alpha - \alpha_o) = n_1 \lambda$$

b (β - β_o) = $n_2 \lambda$

$$b (\gamma - \gamma_o) = n_3 \lambda$$

Here n_1 , n_2 , n_3 **are the integers, form the orders of diffraction along three axes. These are called Laue Equations. These relations represent the essential features of the theory of space lattice.**

Q.3) Explain Bragg's Law of X-ray diffraction or Derive Bragg's condition for X-ray diffraction.
A) Bragg's Law: This Law states that "The Diffraction Maxima occurs when X-rays constructively interfere as they pass through the atomic lattice planes in crystals". Also it can

be stated as: X-rays diffract from crystal lattice planes when the interplanar spacing in it is of the same order as that of wavelength of the incident X-rays.

Derivation: Let us consider a set of parallel lattice planes of a crystal separated by a distance 'd' apart. Suppose a narrow parallel beam of X-rays of wavelength λ be incident upon these planes at an angle ϴ as shown in the figure below:
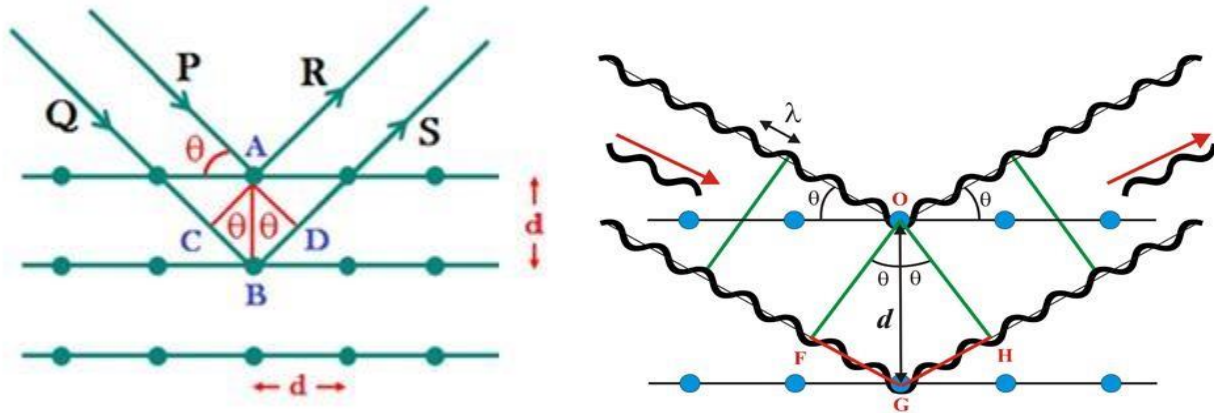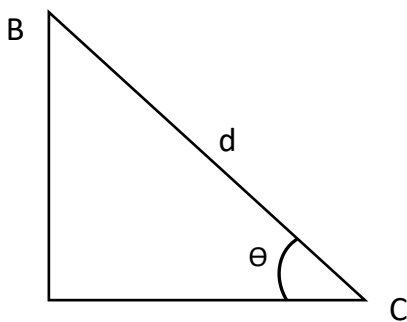

The beam will get reflected in all directions by the atoms of various atomic planes. As the refractive index of the crystal is nearly unity, hence practically there will not be any kind of bending of x-rays entering or leaving the crystal. The condition for the reflected waves to be all in phase is that the path difference between two reflected rays must be an integral multiple of the wavelength incident.

Consider a ray PA incident at an angle ϴ on the atom at A, reflected in the direction of the ray AR. Also another ray QB is reflected at another atom B in the direction BS. Now from the atom A let us draw a normal/perpendicular AC onto the incident ray QB and as well another normal AD onto the reflected ray BS respectively. Now the Path difference between these two rays is (CB+BD).

$$CB+BD = n\lambda \quad \text{--------(1)}$$

From the figure, we have one of the right angled triangle ACB which gives:

$$\text{Sin } \theta = \frac{CB}{AB}$$

=> CB = AB Sin ϴ = d Sin ϴ   (∵ AB = d)

Similarly BD = AB Sin ϴ = d Sin ϴ

A   Hence

CB+BD = nλ

=> d Sin ϴ + d Sin ϴ = nλ

d Sin ϴ = nλ

Where n=1, 2, 3, 4…………. etc for first order, second order, third order ………… etc maxima respectively. This relation is known as the Bragg's relation or Bragg's Law or Bragg's Condition.

Q.4) Explain the concept of Reciprocal Lattice or What is Reciprocal lattice? Explain

A)   Definition: The reciprocal space lattice is a set of imaginary points constructed in such a way that the direction of a vector from one point to another coincides with the direction of a normal to the real space planes and the separation of those points (absolute value of the vector) is equal to the reciprocal of the real interplanar distance.

The notion/concept of reciprocal vectors was introduced in vector analysis by J. W. Gibbs in 1881, in his book, *Elements of Vector Analysis.* This was adapted by P. P. Ewald to interpret the diffraction pattern of an orthorhombic crystal (1913) in his famous paper where he introduced the sphere of diffraction. It was extended to lattices of any type of symmetry by M. von Laue (1914) and Ewald (1921). The first approach to that concept is that of the system of *polar axes*, introduced by Bravais in 1850, which associates the direction of its normal with a family of lattice planes.

The reciprocal lattice is therefore an essential concept for the study of crystal lattices and their diffraction properties.

Definition

Let a, b, c be the basic vectors defining the unit cell of the direct lattice. The basic vectors of the reciprocal lattice are defined by: *

$$a* = \frac{(b \wedge c)}{(a, b, c)} \quad b* = \frac{(c \wedge a)}{(a, b, c)} \quad c* = \frac{(a \wedge b)}{(a, b, c)} \qquad (2.1)$$

The modulus of a* is equal to the ratio of the area of the face *OBCG* opposite to a to the volume of the cell built on the three vectors a, b, c. Referring to Fig. 1, we may write:

$$a* = 1/OA' \quad b* = 1/OB' \quad c* = 1/OC' \qquad (2.2)$$

Fig. 1
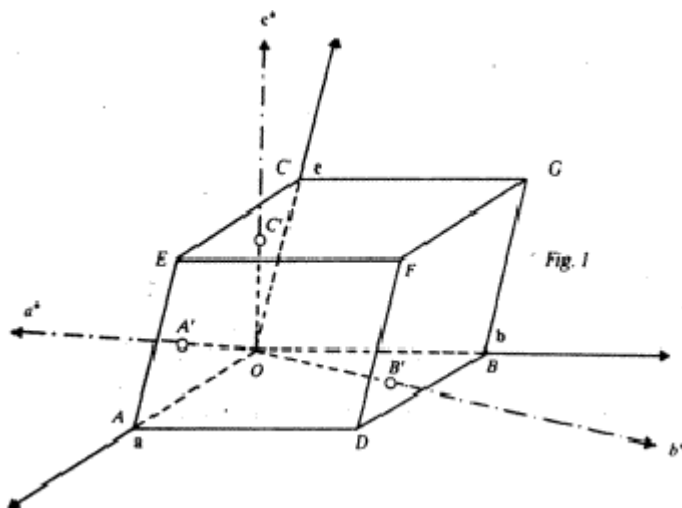
From the definition of the reciprocal lattice vectors, we may therefore already draw the following conclusions:

(i)   Each of the three vectors a*, b*, c* is normal to a set of lattice planes of the direct lattice (b, c; c, b; a, b) and their moduli are respectively equal to the inverse of the spacings of these three sets of lattice planes. The basic vectors of the reciprocal lattice possess therefore the properties that we were looking for in the introduction. We shall see in the next section that with each family of lattice planes of the direct lattice a reciprocal lattice vector may be thus associated.

(ii)   The dimensions of the moduli of the reciprocal lattice vectors are those of the inverse of a length. For practical purposes the definition equations (2.1) may be rewritten after the introduction of a scale factor σ which has the dimension of an area:

$$a^* = \frac{(b \wedge c)}{(a,\ b,\ c)} \sigma \quad (2.3)$$

This is only done to give the reciprocal lattice vector the dimension of length when one wants to actually draw the reciprocal lattice and we shall not make use of this scale factor in this paper.

From relations 2.1 it can readily be shown that the two sets of basic vectors satisfy the following equations:

$$a \cdot a^* = 1 \quad etc\ldots \qquad a \cdot b^* = 0 \quad etc\ldots \hspace{3cm} (2.4)$$

The two sets of equations (2.1) and (2.4) are equivalent and equations (2.4) are sometimes used as the definition equations of the reciprocal lattice. These relations are symmetrical and show that the reciprocal lattice of the reciprocal lattice is the direct lattice.

UNIT – V 7 SUPERCONDUCTIVITY

Q.1) Explain the concept of Superconductivity (or) What is Superconductivity? What are the various experimental facts associated with them?

A)   Definition: The phenomenon in which a substance loses its electrical resistance almost completely i.e., a current flows continuously through it without altering its value is known as Superconductivity. Also in other words, "When the electrical resistivity of a substance falls suddenly to zero, when cooled below a certain temperature" is termed as Superconductivity.

 The substances showing this property are called Superconductors. Substances which show Superconductivity include: Mercury, Cadmium, Silver, Lead, Gallium, Iridium, Tungsten etc.

Following are some of the experimental facts of Superconducting materials:

(i)   At room temperature, superconducting materials have greater resistivity than other elements.

(ii)   The transition temperature T_c is different for different isotopes of an element. It decreases with the increasing atomic weight of isotopes.

(iii) The superconductivity of a material is not lost by adding impurities to it but by lowering the critical temperature.

(iv)   There is no change in the crystal structure as revealed by X-ray diffraction studies. This points that superconductivity is more concerned with conduction electrons.

(v)   Superconductor is characterized by zero electrical resistance. It can conduct current even in the absence of an applied voltage and this current can persist for years without any prominent loss.

(vi)   The thermal properties like entropy, heat capacity, thermal conductivity etc of a metal change sharply at transition temperature of superconductivity as the temperature is lowered. Superconducting state is more orderly state because the entropy decreases as one passes from normal conducting state to superconducting state.

(vii) The elastic properties do not change in transition.

(viii) All thermoelectric effects disappear during superconducting state.

(ix)   At a temperature below T_c, there exists a magnetic field called Critical magnetic field H$_C$ where at which the superconductor loses its superconducting property. The value of H$_C$ decreases as the temperature increases.

Q.2) Explain Meissner Effect (or) What is Meissner Effect? Explain.
A)   Meissner Effect: The expulsion of magnetic lines of force from a superconducting material when it is cooled below the transition temperature in a magnetic field is called Meissner Effect.

Meissner in 1933 observed that if a superconductor is cooled in the presence of an external magnetic field, below the critical temperature corresponding to that field, then the lines of force are expelled from the material. This phenomenon is known as Meissner Effect. Figure (a) below shows the superconductor in normal state with lines of force penetrating/passing through it, while Figure (b) below shows the superconductor in a state where the lines of force expelled out of it when cooled below the transition or critical temperature.
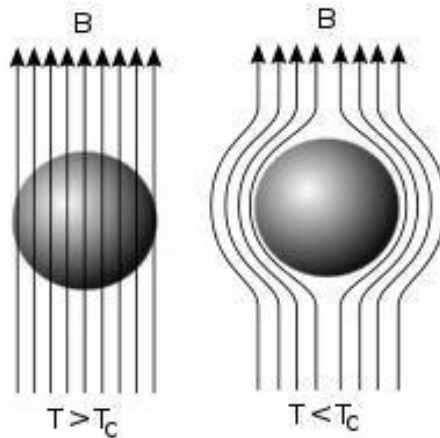

Diagram of the Meissner effect. Magnetic field lines, represented as arrows, are excluded from a superconductor when it is below its critical temperature.

   Important Points:

1) Meissner Effect is Reversible. When the temperature is increased below T_c , the flux suddenly passes through the specimen and the substance comes to its normal state. 2) A Superconductor is a perfect Diamagnetic. The reason is that the magnetic induction B in a Superconductor is zero.

   Therefore for Superconductors,

$$B = \mu_0 (H + M) = 0 \quad \text{or} \quad M = -H, \qquad \qquad \text{...(1)}$$

   H = magnetizing field intensity

and M = intensity of magnetization

The magnetic susceptibility χ is given by

$$\chi = \frac{M}{H}. \qquad \qquad \text{...(2)}$$

From eqs. (1) and (2), we get

$$\chi = -\frac{H}{H} = -1. \qquad \qquad \text{...(3)}$$

   This is the maximum value for the susceptibility of a diamagnetic material. In this sense, a superconductor is a perfect diamagnet.

   (3) Maxwell's equation is given by

$$\vec{\nabla} \times E = -\frac{\partial B}{\partial t} \qquad \qquad \text{...(4)}$$

According to Ohm's law,

$$V = I R \quad \text{and} \quad E = V/d$$

$$\therefore \quad E = \frac{I R}{d} = \frac{(J A) R}{d} = J\left(\frac{A R}{d}\right)$$

or

$$E = J\rho, \quad \text{where } \rho = \left(\frac{A R}{d}\right). \qquad \qquad ...(5)$$

For finite J and zero ρ, E should be zero.

From eq. (4), $\quad \dfrac{\partial B}{\partial t} = 0 \quad$ or $\quad B = \text{constant.}$

Thus for a Superconductor, the condition of zero resistivity predicts that magnetic flux should remain constant, which is in contradiction to Meissner Effect. Thus Superconductors should be judged by both conditions independently. So for a Superconductor,   1) Has Zero Resistance below Critical Temperature   2)

Shows Meissner Effect below Critical Temperature

Q 3) What is Isotope Effect? Explain.

A)   It has been observed that critical temperature (T_C) of semiconductors varies with the isotopic mass. More precisely, transition temperature is found to vary as the square root of atomic mass M as $T_C \propto M^{-1/2}$ or $T_C\, M^{1/2} = $ a constant.

For example T_C for mercury varies from 4.185 K to 4.16 K when M changes from 199.5 to 203.4 amu.

Fig. (5) shows the experimental results for tin. The equation $T_C\, M^{1/2} = $ constant suggests that lattice vibrations are involved in causing superconductivity.

At very high frequencies the zero resistance of the superconductor is modified. Upto 10^7 c/s, as London predicted, it has considerable value even below critical temperature.

Fig. (5) Isotope effect for tin metal

However, it is observed that transition temperature remains unaffected by the frequency.
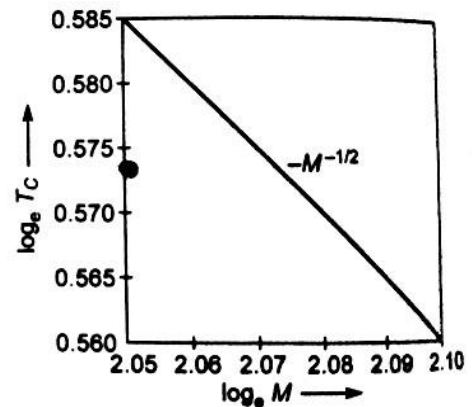
Q 4) What is Superconductivity? Explain Type-I and Type-II Superconductors.

Based on magnetic behaviour, the superconductors are classified into following two categories :
(1) Type-I superconductors or soft superconductors.
(2) Type-II superconductors or hard superconductors.

Type-I Superconductors

The dependence of magnetization of a superconductor of type-I as a function of external field H is shown in fig. (6). It is obvious from the figure that upto the critical field strength (H_C), the magnetization of the superconductor grows in proportion to the external field. As soon as the applied field H exceeds H_C, the magnetization abruptly drops to zero.

So, *type-I superconductor is one in which the transition from superconducting state to normal state in presence of magnetic field occurs sharply at the critical value H_C.*

In presence of an external magnetic field $H < H_C$, type-I superconductor in superconducting state is a perfect diamagnet. When H exceeds H_C, the superconductor enters the normal state, *i.e.*, it loses its diamagnetic property completely. In this state, the magnetic flux
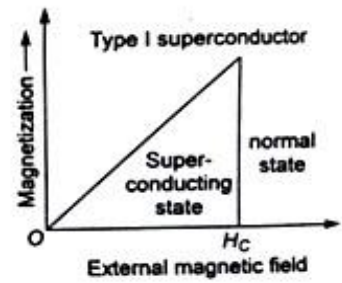

Fig. (6) Showing magnetization curve for a type-I superconductor

A)

penetrates throughout the Superconductor

Type-II Superconductors

The magnetization curve of type-II superconductor is shown in fig. (7). The type-II superconductor is characterized by two critical magnetic fields H_{C_1} and H_{C_2}. The description of the curve is as follows :

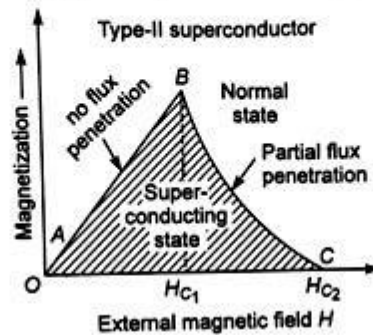(i) For the field strength below H_{C_1}, the superconductor expels the magnetic field from its body

Fig. (7) Showing magnetization curve for type-II superconductor.

Completely and behaves as a perfect diamagnet. Here H_{C_1} is called Lower Critical Field The Curve is represented by AB.

(ii) As the magnetic field increases from H_{C_1}, the magnetic field lines begin to penetrate the material. The penetration increases until H_{C_2} is reached. H_{C_2} is called the upper critical field. At H_{C_2}, the magnetization vanishes completely, *i.e.*, the external field has completely penetrated into superconductor and destroyed the superconductivity.

In region from H_{C_1} to H_{C_2}, the specimen assumes a complicated mixed structure of *normal* and *superconducting* states. The superconductor is said to be in a mixed state which is commonly known as *vortex state*. In the region H_{C_1} and H_{C_2}, the material is in a magnetically mixed state but electrically it is a superconductor.

(iii) After H_{C_2}, the material turns to normal state.

So, *type-II superconductor is one which is characterized by two critical fields H_{C_1} and H_{C_2} and transition to normal state takes place gradually as magnetic field is increased from H_{C_1} to H_{C_2}.*

Let us consider the state of superconductor between H_{C_1} and H_{C_2}. The state is called as mixed state or vortex state. In this state, though there is a flux penetration, yet the material retains its zero resistance properly and it is still a superconductor. The flux penetration takes place through filaments (small channels) as shown in fig. (8). The small channels are the bits of material which has turned to normal state from superconducting state. The number of filaments increases as the value of $H > H_{C_1}$ increases upto H_{C_2}. At H_{C_2}, the filaments spread into the entire superconductor and the material becomes a normal conductor.
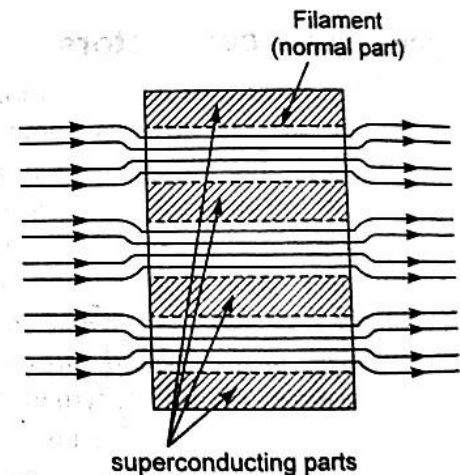
Fig. (8) Mixed state or vortex state.

The most important advantages of type-II superconductor is the value of critical field H_{C_2} which is many more times (even 100 times) higher than the value of H_{C_1} for type-I superconductor. So, the most important use of type-II superconductor is to build up a device which can work in high magnetic fields such as superconducting magnets.

Q 5) What are the Applications of Superconductivity? Or Explain various Applications of Superconductors.

A) The Superconductors find many practical applications. Few of them are as follows:

1) <u>Power transmission</u>: Electrical powe transmisssion through any conductor is

always accompanied by energy loss I^2R, where I is the current and R is the resistance of the conductor. If Superconductors are used, the losses will be eleminated and power transmission can be done at a lower voltage level.

2) <u>Superconducting Magnets</u>: An electromagnet that is made by using coils of Superconducting wires or cables is called Superconducting magnet. The main advantage of these is that once the current is setup, the coil requires no source of EMF to drive the current. Among many the most commonly used superconductor for making these is Niobium Titanium (Nb-Ti), a Type-II superconductor. Also these magnets are used in Magnetic Resonance Imaging (MRI), technique to generate images of various cross-sections of organs & even for entire body.

3) <u>Electrical Appliances (Cryotron)</u>: This consists of a wire of Superconducting material A (Tantalum with Transition Temperature=4.4 K) around which another wire of superconducting material B (Niobium with Transition Temperature=9.3 K) wound in the form of a solenoid as shown in the figure below: The wire A is called as Gate..
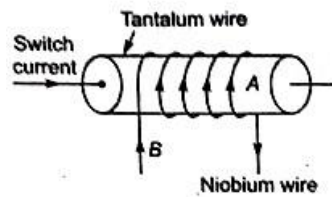
Fig   Cryotron

The cryotron is based on the   principle   of disappearance of superconductivity above H_C. At a temperature below 4.4 K, both A and B are in superconducting state. Allowing the current and switching the magnetic field A and B can be varied so as to operate as a fast on and off switch.

4) <u>Mag-Lev Vehicles</u>: In a Superconductor, the magnetization is in the direction opposite to that of external applied magnetic field. This is known as Diamagnetism. But when a superconductor is brought a strong permanent magnet, there is a strong repulsion between them. This force causes the lighter one to float over the other. This is known as Magnetic Levitation.

The Maglev vehicle (say a train) consists of superconducting magnets built in its base. Say the vehicle runs over an aluminium track in which a current is flowing. The train is set afloat by magnetic levitation as shown in fig. (14). This is due to enormous repulsion between two highly powerful magnetic fields, one produced by the superconducting magnet inside the train and the other due to electric currents in the aluminium track.

5) <u>Very Strong Magnetic Fields</u>: Very Strong Magnetic Fields of the order of 50 Tesla can be generated with the coils made of superconducting materials, which are mostly used in research and medical diagnostics. The cost of these less than the conventional electromagnets.

6) <u>SQUIDS (Superconducting Quantum Interference Devices)</u>: These are fundamentally Superconducting Rings that act as storage deives for magnetic flux. These are used to find the minute changes in the magnetic field of human brain or body.

7) <u>**For Computer Technologies**</u>**: At present there is a persistance of heat losses in the components that can be mounted on a chip, as a result of which there arises a limitation to their use. Usage of superconductors can make sure of greater usage of the components due to elimination of losses.**

<div align="center">**From Unit-I : Atomic and Molecular Physics <u>Essay</u>**</div>

<u>Questions</u>

01. Describe the Stern and Gerlach experiment and discuss how it verifies the concept of Space quantization and electron spin. (OR) Describe the Stern and Gerlach experiment and indicate the importance of the results Obtained. VVIMP

02. What is Raman Effect? Describe experimental arrangement to study Raman Effect. VVIMP

03. Explain various Quantum Numbers associated with the Vector Atom Model.

<u>Short Answer Questions</u>

01. Explain various Coupling Schemes. (Or) What are L-S and J-J Couplings? Explain.

02. Explain Quantum theory of Raman Effect.

03. Write the Applications of Raman Effect or Give the Applications of Raman Effect. 04. Problem on Raman Effect

-- ----------

From Unit – II : MATTER WAVES & UNCERTAINITY PRINCIPLE

Essay Questions

01. Explain de-Broglie Hypothesis of Matter Waves and Derive the expression for de-Broglie Wavelength (or) Describe de-Broglie Hypothesis and Derive the expression for Wavelength of matter waves VVIMP

02. Explain Davisson and Germer Experiment for the detection of Matter Waves (or) Describe Davisson and Germer Experiment on electron diffraction and Discuss its results. VVIMP

03. State & Explain Heisenberg Uncertainty Principle for P and x. Extend it to energy & time

Short Answer Questions

01. Explain the Properties of Matter Waves (or) Give Properties of Matter Waves

02. Problem on Debroglie Wavelength (using various forms)

03. Problem on Heisenberg Uncertainty Principle

From Unit – III : QUANTUM MECHANICS

Essay Questions

01. Derive Schrodinger Time Independent Wave Equation   VVIMP

02. Derive Schrodinger Time Dependent Wave Equation   VVIMP

03. Derive the energy Eigen values and Eigen functions for a Particle in a One Dimensional Potential Well or Box (Or) Obtain an expression for the Energy of a Particle in a Box

Short Answer Questions

01. Write the Physical Significance of Wave function ψ

02. Explain Eigen functions and Eigen values

03. Write the Postulates of Quantum Mechanics

From Unit – IV : 4 GENERAL PROPERTIES OF NUCLEI & 5 RADIOACTIVITY Essay Questions

01. Explain various Properties of Nucleus

02. Explain the Liquid drop model of Nucleus   VVIMP

03. Explain the Shell model of Nucleus   VVIMP

04. **Explain Gamow's theory of α-Decay   VVIMP**

Short Answer Questions

    04. Problem on Nuclear radius or size

    05. Explain Geiger-Nuttal Law

From Unit – V : 6 CRYSTAL STRUCTURE

Essay Questions

    01. Explain fourteen Bravais lattices with neat diagrams (or) How are crystals classified? Explain with neat diagrams all Bravais lattices.

    02. What are Miller Indices? How are they calculated? (or) What are Miller Indices? How they are determined for a crystal plane?

    03. Explain Laue's method of X-ray diffraction for the determination of Crystal structure.

Short Answer Questions

01. **Explain Bragg's Law of X-ray diffraction or Derive Bragg's condition for X-ray diffraction.**

02. **Problem on Bragg's Law**

03. **Problem on Miler Indices**

From Unit – V : 7 SUPERCONDUCTIVITY

Essay Questions

    01. Explain the concept of Superconductivity (or) What is Superconductivity? What are the various experimental facts associated with them?

    02. What is Superconductivity? Explain Type-I and Type-II Superconductors.

    03. What is Superconductivity? Explain various Applications of Superconductors.

Short Answer Questions

    01. What is Meissner Effect? Explain.

    02. What are the Applications of Superconductivity?

    03. What is Isotope Effect? Explain.